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A NEW TECHNIQUE FOR MEASURING EXTRAGALACTIC DISTANCES[a)]

JOHN TONRY[b)]
Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

DONALD P. SCHNEIDER
Institute for Advanced Study, Princeton, New Jersey 08540
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ABSTRACT

We describe a relatively direct technique of determining extragalactic distances. The method relies on measuring the luminosity fluctuations that arise from the counting statistics of the stars contributing the flux in each pixel of a high-signal-to-noise CCD image of a galaxy. The amplitude of these fluctuations is inversely proportional to the distance of the galaxy. This approach bypasses most of the successive stages of calibration required in the traditional extragalactic distance ladder; the only serious drawback to this method is that it requires an accurate knowledge of the bright end ($M_V \leqslant 3$) of the luminosity function. Potentially, this method can produce accurate distances of elliptical galaxies and spiral bulges at distances out to about 20 Mpc. In this paper, we explain how to calculate the value of the fluctuations, taking into account various sources of contamination and the effects of finite spatial resolution, and we demonstrate, via simulations and CCD images of M32 and N3379, the feasibility and limitations of this technique.

I. INTRODUCTION

A basic problem in astrophysics is the measurement of the distance to celestial objects. Truly reliable distances can be derived from radar timing in the solar system and parallax measurements of nearby stars. For more distant sources, we must rely on indirect techniques such as the moving cluster method, statistical parallaxes, or main-sequence fitting (see, for example, Mihalas and Binney 1981). The traditional approach for extragalactic measurements is to construct a distance ladder of ever more luminous and rare beacons that can be seen to enormous distances, stand out from the surrounding background, and which, we trust, have constant or calibratable luminosities. As we bootstrap ourselves up this distance ladder, however, each successive rung is progressively more uncertain; the estimators become more exotic and less well understood, and the propagation of systematic errors produces large cumulative uncertainties after just a few steps.

A different approach is suggested by Fig. 1 [Plate 33]. This high-signal-to-noise picture of the nearby elliptical galaxy M32 reveals a striking mottling that is reminiscent of Baade's (1944) classic resolution of scattered, very bright stars in M31. It is easy to show, however, that, unlike Baade, we are seeing tens of stars per square arcsecond that lie on the ordinary giant branch rather than individual, extremely luminous supergiants. We were therefore led to consider using the amplitude of the mottling as a distance estimator, since it will decrease inversely with distance. The measurement of these "luminosity fluctuations" has the potential to be a very powerful approach, for it bypasses many of the intermediate steps used by previous methods; the only calibration needed is a knowledge of the stellar luminosity function. There are a number of additional advantages to this method, such as high accuracy, ease of observation, and well-understood objects contributing to the luminosity. The primary disadvantages are that the stellar luminosity function must be quite accurately known and that high-signal-to-noise data are required; the latter item demands that the contributors to the noise and fluctuations in the data be well understood. The finite spatial resolution introduced by instrumental and atmospheric blurring, while setting the ultimate limits for the technique, is actually an asset to the method in some respects.

The following section investigates the sources and amplitudes of luminosity fluctuations and describes the steps required for data processing. Section III illustrates the expected behavior of galaxies at different distances, and shows the results of this technique when applied to simulated data. Observations of M32 and N3379 are presented in Sec. IV. In Sec. V we discuss the results and implications, and suggest directions for future research.

II. MEASUREMENT OF LUMINOSITY FLUCTUATIONS

The idea behind luminosity fluctuations is simple. Imagine a galaxy made of only one type of star, ignore seeing for the moment, and suppose that at some region in the galaxy the average projected density of stars is one hundred per pixel. We do not resolve individual stars, but as we look at adjacent pixels we will see fluctuations with rms variations equal to 10% of the mean signal. At different locations in the galaxy, we will see rms fluctuations that vary as the square root of the local mean galaxy brightness; the proportionality constant between fluctuation rms and square root of the mean surface brightness will be directly related to the number of stars present. If the intrinsic luminosity of the stars is known, it is an easy matter to calculate the distance of the galaxy from the measured fluctuations and the observed surface brightness. Now consider an identical galaxy that is twice as distant. If we look at the first location where there used to be one hundred per pixel, there are now 400 stars per pixel because the pixel subtends 4 times the metric area (the flux from each star, of course, is decreased by a factor of 4, so the surface brightness is constant). In this galaxy, the fluctuations are 5% at this flux level. Different regions in the galaxy will have a linear relation between the rms fluctuations

[a)] Observations taken at the Palomar Observatory, California Institute of Technology, and at the McGraw Hill Observatory, operated jointly by the University of Michigan, Dartmouth College, and the Massachusetts Institute of Technology.
[b)] Alfred P. Sloan Fellow.

and square root of galaxy flux, but the proportionality constant will be half as large as for the first galaxy; this constant is inversely proportional to distance.

a) Sources of Fluctuations

In reality, there are many sources of pixel-to-pixel fluctuations in a CCD image of a galaxy. Some, like globular clusters, dwarf companions, or the fluctuations that we seek to measure and employ, are essentially intrinsic to the galaxy in question. Others involve foreground stars or background galaxies, and yet others arise from various kinds of instrumental noise. The total variance (or mean-square value) at any point in the image will be the sum of the variances of each component, under the reasonable assumption of uncorrelated phases.

We will shortly catalog and characterize these wanted and unwanted variances, but already there is one vital point to keep in mind: the spatial power spectrum of fluctuations is the key to this method. First, it will allow us to determine the amplitude of fluctuations that have been degraded by atmospheric blurring. Second, it will give us the means to distinguish many instrumental sources of noise (which have a white power spectrum) from fluctuations intrinsic to the galaxy (which will have the point-spread function (psf) impressed upon them). If we restrict ourselves to early-type galaxies and zealously excise point sources, we can avoid fluctuations from clumping of stars, from patchy obscuration, or from objects such as globular clusters, H II regions, planetary nebulae, companion galaxies, etc. The feasibility of this will be demonstrated below.

First we need some nomenclature. Let us assume that we have a CCD picture of a galaxy with $l \times l$ pixels that has had any bias level subtracted and has been divided by a suitably normalized flatfield exposure, but without any removal of a mean sky brightness. Let $\bar{c}(x,y)$ be the mean total signal at the point (x,y) in ADU (analog–digital unit, an abbreviation used for observed CCD counts), where the mean is taken over many realizations of the galaxy and observation in principle, and is a suitable local fit in practice. Let $\bar{g}(x,y)$ be the mean signal from the galaxy in ADU at (x,y) and use s to denote the sky flux so that $\bar{c} = \bar{g} + s$. Define $a(x,y)$ to be the (inverse) CCD gain in units of electrons per ADU, and m_1 to be the magnitude of an object in the chosen photometric band that yields 1 ADU per second for the instrument and telescope used. (Before flattening, $a(x,y)$ is a constant and m_1 varies as a function of position from vignetting and variations in quantum efficiency.) The exposure time of the observation is t, and d represents the distance to the target galaxy.

The two most obvious (and easy to treat) sources of fluctuations are the readout noise of the device photon shot noise. Expressions for the variance of these features are

$$\sigma_R^2 = a^{-2} N_R^2, \tag{1}$$

where N_R is the CCD readout noise (in electrons), and

$$\sigma_p^2 = a^{-1} \bar{c}. \tag{2}$$

Two other items to be considered are cosmic rays and any noise arising from the counting statistics in the flatfield. These have a negligible effect once the prominent cosmic rays are removed and if flatfields of sufficiently high signal-to-noise are taken, requirements that are easily satisfied in practice. All of these sources of noise have a white power spectrum, and can therefore be distinguished from fluctuations due to the stellar component of the target galaxy.

More troublesome fluctuations are produced by other objects in the field (such as stars, faint background galaxies, globular clusters, etc.) because these sources have nearly the same power spectrum as the luminosity fluctuations we wish to measure. An object whose total flux is f ADU and whose surface number density is n objects per square pixel contributes an amount nf^2 to the variance. This must be integrated over all types of objects that are not excised from the image. (In practice, the variance is measured from the power spectrum of the whole image, so we can assume Gaussian statistics even though $n \ll 1$ per pixel.)

Assume that the number density of faint objects follows a simple power law:

$$N(>f) = (f/f_0)^{-\alpha} \tag{3}$$

(f_0 and α must be empirically determined for each field). If all objects brighter than some limiting flux ($f_{\max}$) are removed from the data, then the fluctuations produced by the remaining sources and (before any blurring from a point-spread function)

$$\sigma_s^2 = \int_0^{f_{\max}} n(f) f^2 \, df = \frac{\alpha}{2-\alpha} f_0^\alpha f_{\max}^{(2-\alpha)}. \tag{4}$$

The power spectrum of these sources will be identical to the psf (to the extent that they are unresolved).

Globular clusters have a Gaussian-shaped luminosity function that peaks at $M_V = -7.11$ and has a Gaussian width of 1.35 mag (van den Bergh 1985). Roughly speaking, globular clusters have a number density proportional to the surface brightness of the galaxy and appear with a specific frequency of 4–10 clusters per $M_V = -15$ of the parent galaxy. Using the assumption that there is one globular cluster associated with each M_{G1} of galaxy luminosity, the number density of globular clusters is

$$n_{GC} = (d/10\,\text{pc})^2 (\bar{g}/t) 10^{+0.4(M_{G1} - m_1)}. \tag{5}$$

If all globular clusters have the same absolute magnitude M_{GC}, the variance from these objects is (ignoring the psf for the moment)

$$\sigma_{GC}^2 = (10\,\text{pc}/d)^2 \bar{g}\, t\, 10^{-0.4(2M_{GC} - m_1 - M_{G1})}. \tag{6}$$

The net power spectrum will be the power spectrum of a globular cluster convolved with the psf; for galaxies at distances greater than 5 Mpc, this is virtually identical to the psf.

The variance of the luminosity of a stellar population is easily derived if we know the types of stars that comprise the luminosity function. If the population consists of various species with luminosity L_i and expectation number of stars n_i, the expectation value of the luminosity is

$$\langle L \rangle = \sum n_i L_i, \tag{7}$$

and the variance (in the limit of large n_i) is

$$\langle (L - \langle L \rangle)^2 \rangle = \sum n_i L_i^2. \tag{8}$$

We define a mean, luminosity-weighted luminosity of the population as

$$\bar{L} \equiv \sum n_i L_i^2 / \langle L \rangle. \tag{9}$$

We can now derive the variance in ADU at a given point from these luminosity fluctuations (if there were no atmospheric blurring) as

$$\sigma_L^2 = \bar{L}\langle L \rangle = \bar{g}(x,y)t(10\ \text{pc}/d)^2 10^{-0.4(\bar{M}-m_1)}. \quad (10)$$

Note that σ_L^2 increases as $t^2(\bar{g} \propto t)$; although these fluctuations superficially appear to be "noise," they do in fact behave like "signal." We have assumed here that the number of stars per pixel n_i is large enough to use Gaussian rather than Poisson statistics for the variance. The validity of this assumption will be demonstrated below.

Other sources of variance are negligible. In early-type galaxies, any unbound star clump will rapidly phase mix apart. Fluctuations from dust obscuration are probably not a difficulty, although not enough is known yet about dust in elliptical galaxies to be sure. Dust that occurs on large scales is readily recognizable in some ellipticals (e.g., Ebneter, Djorgovski, and Davis 1988), and the region can be excluded from the analysis. Dust on small scales that originates from evolved stars is seen in 100 μm emission by *IRAS* (Jura *et al.* 1987), but the amount of dust present that produces a typical *IRAS* flux has an optical depth of ≈ 0.005 mag in the r band through the entire galaxy at a radius of R_e; this is not enough to cause a significant amount of variance.

Two final sources of fluctuation are calibration error, where division of the image by the flatfield does not yield a perfectly flat image, and uncertainties introduced by subtracting the mean image during the preparation for the computation of the variance (see Sec. II *b*). The first is likely to be a problem only for fluctuations significantly smaller than 1% of the signal (although it varies with detector), and will have a power spectrum dominated by very low wave numbers. The latter effect is primarily a computational problem, and although the lowest wave numbers will be suspect for any scheme, the fluctuation signal will extend to fairly high wave numbers since it has the power spectrum of the psf.

Armed with the expressions for the variance, we can now investigate the feasibility of this approach. In practice, the fluctuations due to photon shot noise from the sky will dominate the readout noise from the detector. The ratio of photon shot noise to the luminosity fluctuations is

$$\frac{\sigma_p^2}{\sigma_L^2} = \frac{1}{at}\frac{\bar{c}}{\bar{g}}\left(\frac{d}{1\ \text{Mpc}}\right)^2 10^{-0.4(m_1-\bar{M}-25)}. \quad (11)$$

Since $\bar{M} \approx 0$ (the light from ellipticals is primarily produced by K giants; see below), an exposure with a 5 m telescope ($m_1 \approx 25$) will produce luminosity fluctuations greater than the photon shot noise in just seconds at a radius in the galaxy where its surface brightness equals the sky background (Fig. 1 gives a nice empirical confirmation of (Eq. 11)).

A more stringent constraint is contamination by faint objects unassociated with the galaxy.

$$\frac{\sigma_s^2}{\sigma_L^2} = \frac{\alpha}{2-\alpha}\frac{f_0^\alpha f_{\max}^{2-\alpha}}{\bar{g}t}\left(\frac{d}{10\ \text{pc}}\right)^2 10^{-0.4(m_1-\bar{M})}. \quad (12)$$

To make this expression a bit more transparent, let us take $\alpha = 1$ (a typical value, see Sec. IV), let p represent the image scale (in $''$ per pixel), and define m_0' as the magnitude where $N(>f) = 1$ object per square arcminute ($m_0' \approx 19$). Then

$$\frac{\sigma_s^2}{\sigma_L^2} = 0.07\,p^2\frac{f_{\max}}{\bar{g}}\left(\frac{d}{1\ \text{Mpc}}\right)^2 10^{-0.4(m_0'-19-\bar{M})}. \quad (13)$$

Experience shows that it is possible to identify and excise objects with $f > 0.3\,\bar{g}$, so it should be possible to reduce the variance from foreground stars and background galaxies to considerably less than that of the luminosity fluctuations out to a distance of at least 10 Mpc. Measurement of $N(>f)$ for the brighter objects will also provide an estimate of the residual variance from stars.

An integration of the globular cluster luminosity function to a cutoff absolute magnitude M_C gives

$$\frac{\sigma_{GC}^2}{\sigma_L^2} = \frac{1}{2}\,10^{-0.4(2M_{GC}-M_{G1}-\bar{M}-0.8\ln 10\Delta_{GC}^2)} \times \text{erfc}\left(\frac{M_C - M_{GC} + 0.8\ln 10\Delta_{GC}^2}{\sqrt{2}\Delta_{GC}}\right), \quad (14)$$

where Δ_{GC} is the Gaussian width of the luminosity function. Substitution of typical values of $M_{G1} = -13$, $M_{GC} = -7.11$, and $\bar{M} = 0$ indicates that the variance due to globular clusters dominates that expected from the luminosity fluctuations; essentially all of the globular clusters must be removed. This is possible as long as the flux from a globular cluster is larger than the rms background variations. The central surface brightness of a globular cluster of absolute magnitude M_C can be detected at a signal-to-noise SN above the rms luminosity fluctuations at a background surface brightness of

$$\mu_{GC} = 13.7 + 2(M_C + 7) - \bar{M} + 5\log\left(\frac{d}{1\ \text{Mpc}}\right) + 5\log\left(\frac{\text{SN}}{5}\right) + 5\log\left(\frac{\beta}{0.4''}\right), \quad (15)$$

where β is the Gaussian width of the psf. The radius of the isophote at this surface brightness is defined as R_{GC}. During the data processing, a value of M_C is selected to make $\sigma_{GC}^2 \ll \sigma_L^2$. Combining this with the seeing profile and an estimate of the distance to the galaxy yields the limiting radius R_{GC}; the region of the galaxy interior to R_{GC} is then ignored in the analysis.

b) Data Processing

In practice, how does one go about recovering and measuring the fluctuation signal from a CCD observation of a galaxy? There are essentially three steps: (1) prepare the frame by identifying and excising cosmic rays, bad columns, saturation tracks, and all point sources that could disturb a fit of the local mean; (2) derive a smooth local mean so that the variance about this mean can be calculated, subtract the fit, and remove the remaining point sources; and (3) Fourier transform the image and fit the power spectrum as a sum of the psf power spectrum and a constant. Each of these steps will be discussed in detail, and they will be applied to observations of M32 and NGC 3379.

To compute the variance of the galaxy luminosity about the mean, it is necessary to subtract the mean luminosity quite accurately, because over much of the galaxy the surface brightness changes by an amount comparable to the fluctuations on a scale length typical of the seeing disk. This requires first removing all defects and objects that will disturb a fit to the mean profile, and then fitting a galaxy profile of ellipses of arbitrary radius, eccentricity, and position angle. From this edited picture, a smooth image is constructed and subtracted from the original data. Generally, this does not produce a perfect fit because of non-elliptical isophotes, nonconcentric isophotes, or parts of other galaxies in the field. The residual image must be smoothed on a scale approximately ten times the size of the psf and then subtracted from the original image. This produces an image with zero mean whose fluctuation power spectrum is intact for wave numbers larger than $\frac{1}{3}$ times the width of the psf power spectrum.

With the removal of the galaxy, all point sources brighter than some flux limit can be cataloged and their flux measured. Then, constructing $N(>f)$ (defined as the number of objects per square arcminute brighter than f) from the resulting list, we fit a power law. We select an $f_{\max}$ at which our list of objects is complete, and if the galaxy is near enough this will make σ_s^2 negligible compared to σ_L^2. Each object brighter than $f_{\max}$ is excised, and an estimate of the residual variance from the objects not removed can be calculated.

As described in the previous section, the variance of the fluctuations will be proportional to the (subtracted) mean signal, but we are only interested in the proportionality constant. At this point, we divide by the square root of the mean galaxy signal so that the amplitude of the luminosity fluctuations is constant across the picture and the amplitude of photon shot noise is nearly constant. The main sources of variance now become

$$\tilde{\sigma}_L^2 = t\,(10\ \mathrm{pc}/d)^2 10^{-0.4(\overline{M} - m_1)}, \tag{16}$$

$$\tilde{\sigma}_p^2 = a^{-1}[1 + (s/\overline{g})]. \tag{17}$$

If objects brighter than $f_{\max}$ are excised and the density of objects is uniform across the image, the residual variance in the normalized image from point sources will be

$$\tilde{\sigma}_s^2 = \frac{p^2}{3600\,\overline{g}}\left(\frac{\alpha}{2-\alpha}\right) f_0^\alpha f_{\max}^{2-\alpha}. \tag{18}$$

Next, the radius inside of which globular clusters cannot be reliably identified ($R_{\rm GC}$ from Sec. II *a*) is calculated and data interior to $R_{\rm GC}$ is excised. We can compute an outer radius where the ratio of the signal-to-white-noise error is maximized. By summing pixels within a radius R from the galaxy center, the total variance from the luminosity fluctuations is

$$S = \sum_{<R} \tilde{\sigma}_L^2 \tag{19}$$

and the rms scatter of the total variance from the photon shot noise is

$$N = \left(2 \sum_{<R} \tilde{\sigma}_p^4\right)^{1/2}. \tag{20}$$

This ratio is maximized at a radius $R_{\rm SN}$ that depends on the brightness of the sky background; roughly speaking, this occurs when $\overline{g} = 0.5s$. Data exterior to this radius are excluded from the analysis.

It is simple to understand what sort of power spectrum we should see. The data at this point consist of fluctuations from the various sources multiplied by a window function that is zero where data were excised. Ignoring for the moment the psf, and assuming random phases between the various sources and from pixel to pixel, the resulting power spectrum has a variance at each wave number that is just the sum of the variance at each pixel not masked out. Multiplying by a window function convolves the Fourier transform with the transform of the window, reducing the variance to this limited sum. Some of the fluctuations are actually convolved with the psf, however. In Fourier space, this is just multiplication by the transform of the psf; the variance of these fluctuations will be unaffected at zero wave number but will taper off at high wave numbers just like the psf power spectrum *very* slightly broadened by the window function. Thus, the galaxy fluctuations can be recovered independent of the seeing as the zero wave number limit of the power spectrum, or, for a more reliable measurement, as the factor that must be used to multiply the psf power spectrum (which can be determined very accurately) to match the fluctuation variance. At high wave numbers the instrumental, white-noise contribution will dominate; the seeing provides a nice separation of fluctuations from inside and outside the atmosphere.

The final step is to Fourier transform this image and compute the power spectrum. The same is done for the point-spread function, which is derived from a number of unsaturated stars; its power spectrum is then fitted with a smooth curve. The observed image power spectrum is the sum of the psf power spectrum and a constant:

$$\text{image} = P_0 \times \text{psf} + P_1. \tag{21}$$

This fit is performed from wave numbers not affected by mean subtraction to the Nyquist frequency. We can now relate the coefficients to the quantities of interest,

$$P_0 = \sum \tilde{\sigma}_L^2 + \sum \tilde{\sigma}_s^2 + \sum \tilde{\sigma}_{\rm GC}^2, \tag{22}$$

$$P_1 = \sum \tilde{\sigma}_R^2 + \sum \tilde{\sigma}_p^2, \tag{23}$$

where the sum is over all pixels not excised. P_1 gives us the CCD gain in the limit where the readout noise is negligible. The quantity that we have been seeking, $\tilde{\sigma}_L^2$, which is inversely proportional to the square of the distance of the galaxy, can be calculated from P_0 provided that $\tilde{\sigma}_s^2$ and $\tilde{\sigma}_{\rm GC}^2$ are negligible or can be reliably estimated.

Using data from Gunn, Stryker, and Tinsley (1981) for an old population of stars typical of elliptical galaxies, we derive approximate values of $\overline{L}_V = 58\mathscr{L}_\odot$ or $\overline{M}_V = 0.41$. Clearly, this will depend on the age and metallicity of a stellar population, but the details of how $\overline{M}_V$ varies with $(B-V)$ and Z will be treated elsewhere (Tonry 1988). Almost all the contribution comes from stars at the turnoff up along the giant branch; almost no variance comes from blue stragglers, supergiants, or the main sequence. Low metallicity or young stellar populations will have larger $\overline{L}$, and this is potentially an important effect, although the distances we derive depend on the square root of $\overline{L}$.

III. SIMULATIONS

a) Analytical Estimates

To illustrate these ideas, Table I presents the results expected for an elliptical galaxy at various distances. The galaxy is assumed to have $B_{\rm T} = 10.15$, $(B-V) = 1.00$, and $R_e = 1.0'$. The calculations represent observations in the V filter with a sky surface brightness of 21 V mag arcsec^{-2}. The detector is taken to have 0.4″ pixels, a gain $a = 2\ e^-$ per

TABLE I. Parameters for $V = 9.15$ $R_e = 60''$ galaxy.

Distance (Mpc)	L ($10^{10} L_\odot$)	R_e (kpc)	$\sigma_L(R_{\max})/\overline{c}$ (%)	$R_{\rm GC}$ (″)	$\overline{c}(R_{\max})$ (ADU)	$t_{\rm obs}$ (s)
0.7	0.01	0.20	23	0	22	1.7
1.0	0.02	0.29	16	0	35	2.8
2.0	0.08	0.58	8	2	98	8
4.0	0.30	1.16	4	8	330	26
8.0	1.20	2.33	2	22	1200	96
16.0	4.80	4.65	1	52	4800	380

ADU, readout noise of 8 e^-, and $m_1 = 25$; this corresponds to an overall system quantum efficiency of 10% for a 5 m telescope. The seeing is taken to be 1″.

The radius where the ratio of fluctuation amplitude to overall flux is maximized, $R_{\max}$, occurs where the surface brightness of the galaxy equals that of the sky. (The ratio of luminosity fluctuation amplitude to overall signal goes to zero at large radius because the sky signal overwhelms the galaxy, but the ratio also becomes very low at small radii because the fluctuations go as the square root of the galaxy signal.) The galaxies in Table I are as bright as the sky at $R_{\max} = 0.18\ R_e = 49''$. Since all of the galaxies in Table I have the same observed properties, the radius where the cumulative fluctuation signal relative to white-noise uncertainty is maximized, $R_{\rm SN}$ (see Sec. II *b*), is the same: $R_{\rm SN} = 63''$. Listed in Table I are (1) the distance assumed for the galaxy, (2) the luminosity of the galaxy, (3) the effective radius, (4) the ratio of the luminosity fluctuations to the total signal at $R_{\max}$ (assuming $\overline{M}_V = 0.41$), (5) the radius where a globular cluster of $M_V = -6.11$ is detectable with SN = 5, (6) the value of $\bar{c}$ at $R_{\max}$ for an observation such that the variance from luminosity fluctuations equals that of photon statistics, $\sigma_L^2(R_{\max}) = \sigma_{\rm p}^2(R_{\max})$, and (7) the integration time $t_{\rm obs}$ required to achieve this signal.

The entries in Table I show that it is quite easy to measure the fluctuations and requires but a small amount of telescope time if the distance to the galaxy is less than 4 Mpc. At greater distances, the fluctuations can be determined with increasing difficulty, but the primary limitation is the contamination of the signal by globular clusters. At distances of $\approx$20 Mpc, it is no longer possible to separate the fainter globular clusters from the background variations at radii smaller than $R_{\rm SN}$. Ideally, actual observation times should be longer than those listed in Table I in order to make $\sigma_{\rm p}^2 \ll \sigma_L^2$ to facilitate the identification and removal of point sources.

b) Image Simulations

To quantify the feasibility of this technique as well as monitor the performance of our processing software, we created several frames of simulated data. These pictures were designed to mimic 4-Shooter (Gunn *et al.* 1987) observations in the *r* filter of Thuan and Gunn (1976); readout and photon noise were simulated, as well as representative point-spread functions. The instrument parameters were very similar to those used in Sec. III *a* except that the 4-Shooter has an image scale of 0.335″ per pixel and a total quantum efficiency of $\approx$30%. The galaxies were constructed by randomly selecting stars from a Population II luminosity function (with a very luminous $\overline{M}_r = -2.0$); the normalization was set by the "observed" surface brightness. For a detailed discussion of this software, see Bahcall and Schneider (1987). The pictures ranged from 50 s exposures of "M31" up to 300 s frames of a "Virgo galaxy."

The simulated galaxies had distances of 1, 3, and 10 Mpc. The data were analyzed as described above, and the resulting distances derived were 1.1, 3.1, and 11 Mpc (a truncated psf in the simulations led to the slight overestimate in distance). These simulations demonstrate that the reduction and analysis works, and that accuracies on the order of 10% are easily attainable.

IV. OBSERVATIONS

Despite the relatively small investment in telescope time needed for this technique, images of sufficiently high signal-to-noise ratio are not commonly taken because the central regions of galaxies will saturate a detector many times over in the required exposure time. We present observations of M32 and NGC 3379, located at distances of approximately 0.7 and 10 Mpc, that illustrate the power and difficulties of this method of determining distances.

a) M32

Eight exposures of M32, each of 8 s duration, were taken on 9 January 1984 with the PFUEI (Gunn and Westphal 1981) mounted at the prime focus of the Hale telescope. The detector was an 800×800 Texas Instruments CCD; the readout noise was 10 e^-, and the gain (in the averaged picture) was 8. Reimaging optics produced an image scale of 0.415″ per pixel. The data were taken through the *r* filter (6550 Å) of the Thuan and Gunn (1976) system; observations of standard stars measured a photometric constant of $m_1 = 25.67$. The seeing was moderately good ($\approx 1.3''$), and the sky brightness was 20.52 *r* mag arcsec^{-2} (158 ADU per pixel). This particular CCD has large quantum-efficiency variations ("dark corners"). To correct this, we constructed a sky flat from other observations during the night. This approach produced a beautifully calibrated picture, but at a cost: the sky flat has rms variations of $\approx 1\%$, a value many times that present in the commonly used "dome flats." The averaged picture is displayed in Fig. 1; the data have been stretched to emphasize the mottling in the outer regions of the galaxy.

Reduction of the data for M32 proceeded as described above. The image-processing package VISTA (kindly provided by Tod Lauer) created a smooth fit to the galaxy after bad columns, saturation tracks, cosmic rays, and bright stars were removed. The residual map was then fitted and a zero-mean image was derived by subtracting the smoothed galaxy fit and the residual fit from the original image. Stars were identified in this new image; Fig. 2 shows the $N(>f)$ relation for these data. A power law fit gives $f_0 = 4164$ ADU ($r = 18.9$) and $\alpha = 0.654$.

The effective radius of M32 is 33″ (de Vaucouleurs *et al.* 1976), from which we derive $R_{\rm GC} = 3''$, $R_{\max} = 65''$, and $R_{\rm SN} = 78''$. The central 256×256 of this image was extracted, the nucleus and all stars with $f > 1000$ were excised, the resulting image Fourier transformed, and the power spectrum computed. A bright, unsaturated star was chosen to represent the psf. Using $l = 256$, the following quantities were derived by summing over the extracted, excised image:

$$\sum 1 = 0.970\ l^2,$$

$$\sum \frac{1}{\bar{g}} = 2.48 \times 10^{-3}\ l^2.$$

The first equation shows that 3% of the pixels were excluded from the analysis, while the second equation indicates that the harmonic mean of the surface brightness of the galaxy was 403 ADU per pixel. This flux level implies that an average of about 20 giant stars are present per pixel, which justifies our use of Gaussian instead of Poisson statistics. More distant galaxies, of course, will have far more stars contributing to the flux in each pixel.

Figure 3 reveals that the power spectrum of the image is very accurately described as a constant plus a factor times the power spectrum of the point-spread function. The fit gives

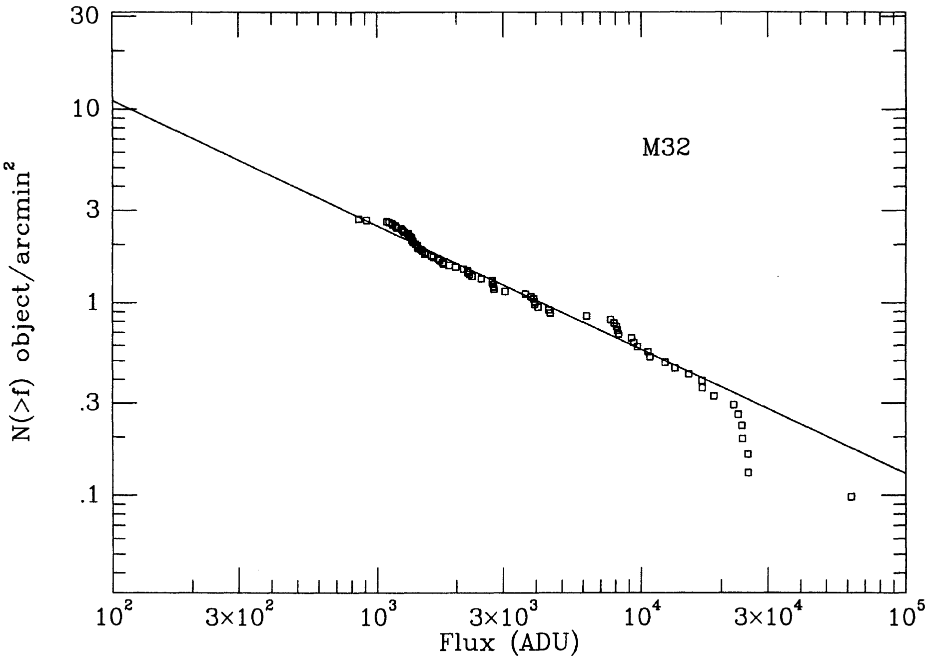

FIG. 2. Plot of the number density of foreground stars/background galaxies in the M32 field as a function of brightness. All objects with a flux greater than 1000 ADU ($r = 20.4$) were removed from the data.

$$P_0 = 15.12\, l^2,$$
$$P_1 = 0.322\, l^2.$$

This, combined with the noise introduced from the sky flat, indicates that the gain was ≈ 7 (compared to an expected value of 8). The extrapolated residual variance from stars, $\tilde{\sigma}_s^2 = 0.146\, l^2$, is negligible compared to the variance produced by the luminosity fluctuations.

The value of $\overline{M}_V = 0.41$ can be converted to the r band using the integrated color of a typical elliptical galaxy ($V - r$ of 0.20). (To do this properly, one should calculate $\overline{M}_r$ using M_V and ($V - r$) for each of the stellar classes that comprise the luminosity function.) Adopting an r band extinction of 0.16 mag towards M32, a value half of the Burstein and Heiles (1978) value of $A_B = 0.32$ mag, we finally derive a distance of 830 kpc for M32.

b) NGC 3379

N3379 was observed in March 1988, using a 584×390 Texas Instruments 4849 virtual-phase CCD (the "Bricc

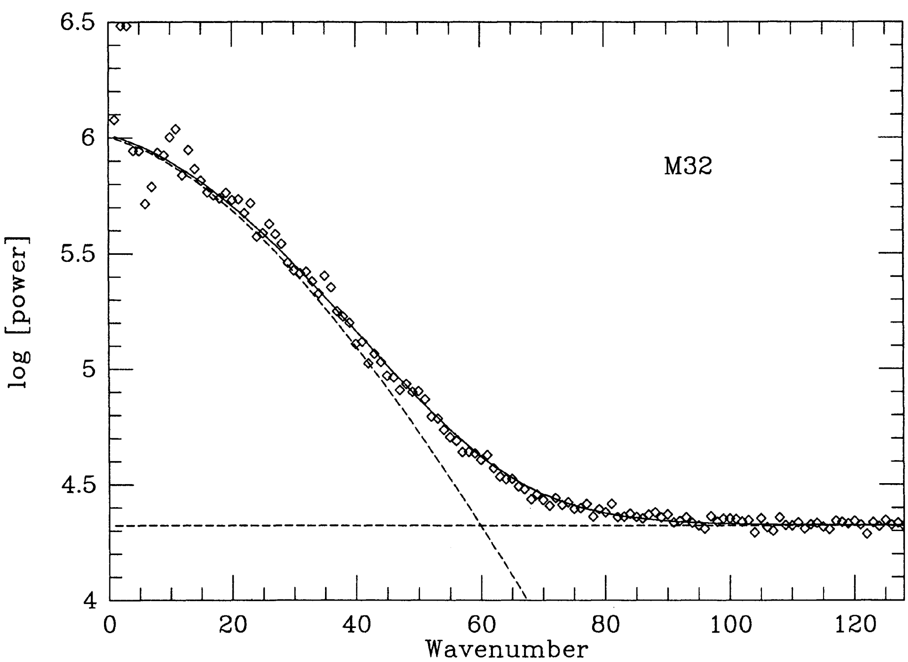

FIG. 3. Power spectrum of the Fourier transform of the processed M32 data. The observations (diamonds) are well fit by the sum of the point-spread function and a constant, shown as dashed lines.

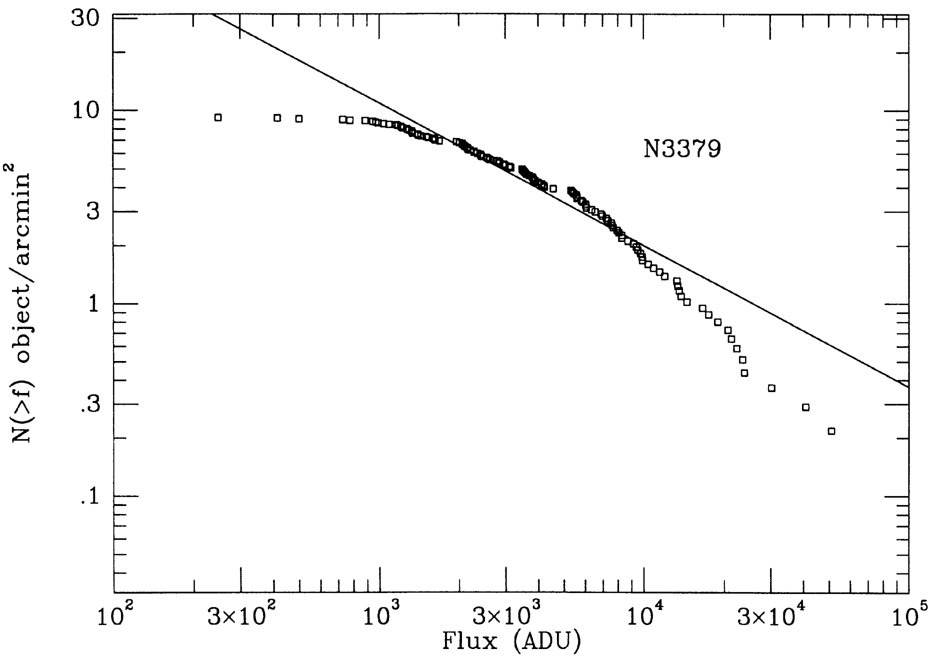

FIG. 4. Number density of globular clusters and foreground stars/background galaxies for the N3379 field. Objects brighter than 300 ADU ($r = 24.6$) were removed from the data. A power law is not a satisfactory representation of the data because most of the objects are globular clusters.

camera," Luppino *et al.* 1987) at the Cassegrain focus of the McGraw–Hill 1.3 m telescope. Images were taken in the r band, with a total exposure time of 3400 s. The CCD had a gain of 4.30 and a readout noise of 8 e^-; the image scale was 0.47″ per pixel and the photometric sensitivity was $m_1 = 21.96$ (r band). Unfortunately, the data were taken in poor seeing (2.2″ FWHM). The sky background contributed 2525 ADU per pixel (20.64 r mag arcsec^{-2}).

The N3379 data were initially processed in the same manner as for M32, removing the mean value of the image. We then identified all globular clusters and stars in the image and constructed $N(>f)$, illustrated in Fig. 4. A power law fit gives $f_0 = 25\,468$ ADU ($r = 19.8$) and $\alpha = 0.736$, but this is clearly a bad representation of $N(>f)$ (see Fig. 4) because the vast majority of point sources in the field are globular clusters. Although not obvious in Fig. 4, the identification is complete to a flux of less than 300 ADU, judged by comparison with Fig. 1 of Pritchet and van den Bergh (1985), which shows the globular clusters they found in a much deeper and sharper CFHT CCD image of N3379. (Essentially all objects visible in their Fig. 1 could be seen in this image.) The effective radius of N3379 is 56″ (de Vaucouleurs *et al.* 1976), from which we derive $R_{GC} = 26''$ (for $M_C = -7.11$ and S/N = 5), $R_{max} = 43''$, and $R_{SN} = 61''$. The flux limit of 300 ADU corresponds to an apparent magnitude of 24.6, and an absolute magnitude of -5.4 at a distance of 10 Mpc. Thus, we expect that the residual variance from stars and globular clusters is negligible with respect to the luminosity fluctuations.

We excised all point sources seen, corresponding to a flux limit of 300 ADU, and also a central region of radius 20 pixels that suffers from poor subtraction and where no globular clusters could be seen. This is considerably smaller than R_{GC} above, but with the external confirmation of Pritchet and van den Bergh, we could confidently work with S/N = 2 detections of point sources. Varying the point-source cutoff to 600 ADU changed the derived distance by only 3%, and increasing the size of the central region excised to a radius of 50 pixels changed the distance by 2%. An area of size 256×256 pixels was extracted, the resulting image Fourier transformed, and the power spectrum computed. A bright, unsaturated star was chosen and its power spectrum was also calculated. Using $l = 256$, the following quantities were derived by summing over the extracted, excised image:

$$\sum 1 = 0.846\, l^2,$$

$$\sum \frac{1}{g} = 4.31\times10^{-4}\, l^2.$$

The power spectrum of the image is shown in Fig. 5, and it is plausibly described as a constant plus a factor times the power spectrum of the point-spread function. The fit gives

$$P_0 = 1.392\, l^2,$$
$$P_1 = 0.654\, l^2.$$

From this we derive a gain $a = 3.2$, which is in good agreement with $a = 4.3$ for an unflattened picture (the flatfield was normalized by the central 100×100 pixels, which has a higher response than the rest of the frame).

Using $\bar{M}_r = 0.21$ and an r band extinction of 0.09 mag towards N3379 (half of the de Vaucouleurs *et al.* (1976) value for $A_B = 0.19$ mag), we derive a distance of 9.8 Mpc for N3379. The mean heliocentric redshift of the Leo cluster, of which N3379 is a member, is around 880 km s^{-1}, and the redshift distance of the Leo cluster is 760 km s^{-1} after correction for motion of the Sun relative to the Local Group and for a Virgo infall of 250 km s^{-1} (Tonry and Davis 1981). We do not presume to divide the two numbers, however, because of the potentially large systematic errors that stellar population age and metallicity could impose on $\bar{M}_r$ and therefore on the distance of 9.8 Mpc.

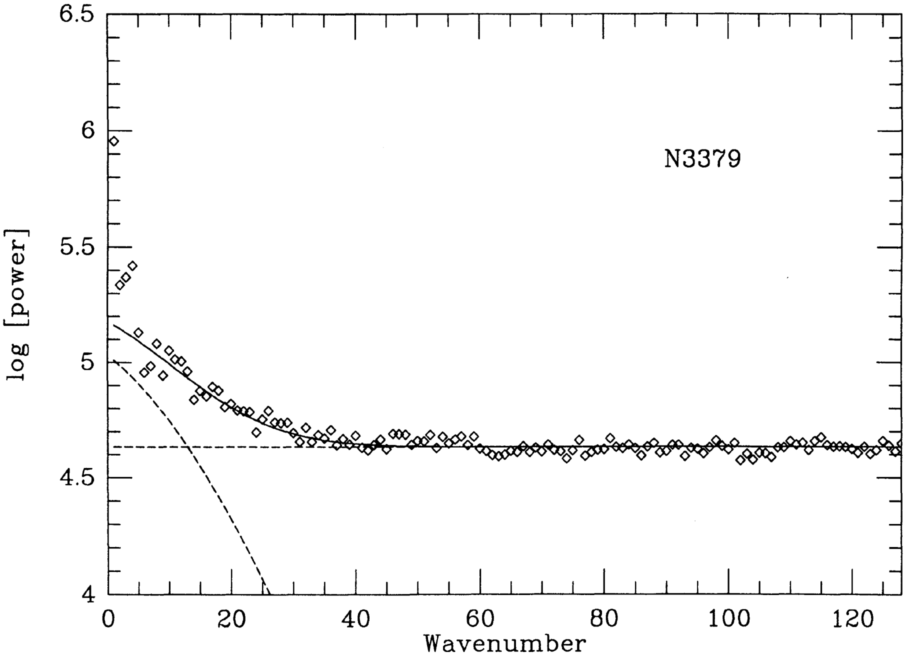

FIG. 5. Power spectrum of the N3379 data. The symbols are the same as in Fig. 3.

V. DISCUSSION

There is little doubt that luminosity fluctuations from the counting statistics of the stars in early-type galaxies must exist. We have demonstrated that they can, in fact, be detected in galaxies closer than roughly 20 Mpc, and can be measured quite accurately. With care, the variance can be determined to better than 10%. By exploiting the spatial power spectrum of various noise terms, it is straightforward to recover the fluctuation signal, even when its amplitude in the image is considerably smaller than that of the white noise present and there is no visible mottling in the image.

In measuring the fluctuation signal, it is important to remove the mean galaxy brightness and all point sources in the image. Using simulations and observations of M32 and N3379, we have demonstrated that this is feasible, even with data that are not of extraordinarily high quality. With a modest investment of telescope time, it is possible to obtain a CCD image with high enough signal-to-noise to permit removal of point sources and measurement of luminosity fluctuations. Again, this is limited to distances of order 20 Mpc for ground-based telescopes.

The goal of this effort is to derive distances from the luminosity fluctuations; distance is inversely proportional to the square root of the fluctuation variance. In order to convert a luminosity fluctuation to a distance, we must know the characteristic luminosity, termed $\bar{L}$ here, of the stellar population in the galaxy. This is simultaneously the real strength and weakness of this method. The stars that contribute to $\bar{L}$ are ordinary giant stars, and, in principle, we can predict $\bar{L}$ from stellar evolution models that only need to be tied to the main-sequence calibration. In one step, we bypass most of the extragalactic distance ladder and directly determine the extragalactic distance scale.

To the extent that we believe that $\bar{L}$ is a universal luminosity, we can immediately derive relative distances between galaxies. However, it is clear that $\bar{L}$ depends on the age and metallicity of a stellar population, with gross variations in age and metallicity (e.g., log [Fe/H] going from -2.3 to $+0.5$) causing $\bar{L}$ to change by as much as an order of magnitude (VandenBerg and Bell 1985). This is potentially a very serious problem, even though we do not expect nearly this sort of variation among large elliptical galaxies, and the distance estimates depend only on the square root of $\bar{L}$. Although the mean color of a population correlates well with $\bar{L}$, it remains to be seen whether the color and line strength of a galaxy can constrain its stellar population well enough to realize the promise of this method. However, we expect that $\bar{M}$ can be determined to at least 0.4 mag, which gives a distance accurate to 20%, offering a considerable improvement over existing measures.

Our observations of M32 and N3379 yield distances of 830 kpc and 9.8 Mpc, respectively. A current determination of the distance of M32 from other methods gives 710 kpc (for M31; Welch *et al.* 1986). The most plausible explanation for the discrepancy is that $\bar{M}$ is 0.34 mag fainter in M32 than we have assumed. M32 is very metal rich for its luminosity (Faber 1973), so this is not necessarily worrisome, but this certainly indicates the importance of a careful study of the metallicity and age of the population. M32 is near enough that it may be possible to observe directly the top of the giant branch, at least in the outer parts of the galaxy.

The redshift distance of N3379, after correction for Virgo inflow with local amplitude 250 km s^{-1}, is 760 km s^{-1}. The Hubble constant is thought to be between 55 and 90 km s^{-1} Mpc^{-1} (Huchra 1987), which implies that N3379 is between 8.4 and 13.8 Mpc distant. The value that we have used for $\bar{L}$ is consistent with this range of distance, the two extremes calling for $\bar{M}_r$ to be 0.32 mag fainter and 0.75 mag brighter than our assumed value. It is obviously premature to calculate a Hubble constant on the basis of this method, but, in principle, it will be possible to determine H_0 from measurements of luminosity fluctuations. A detailed study of galaxies in the core of the Virgo cluster will be very important for this project.

Another exciting use for relative distances derived from luminosity fluctuations is to make an accurate map of the peculiar velocities in the Local supercluster. There has been a recent challenge to the idea that most of the Local Group's peculiar velocity arises locally and from the Virgo supercluster. Lynden-Bell *et al.* (1988) have claimed that a "great attractor" at 4500 km s^{-1} is causing much of the Local Group's peculiar velocity in the direction of Virgo because of the tidal component of its gravitational field. Accurate peculiar velocities in the Local supercluster are still limited by inaccurate distances, and if these distances were known better, it would be possible to distinguish a Virgo infall model from a tidal model, and perhaps even to identify local structure in the peculiar velocity field.

It is also possible to take advantage of luminosity fluctuations when one knows the distance of the galaxy in question. If the galaxy is near enough that point sources can be removed, the luminosity fluctuations can tell us about the characteristic luminosity $\bar{L}$. This will be an interesting probe of $\bar{L}$ as a function of metallicity and age of a galaxy, and can even be used to determine gradients in stellar populations as a function of radius, or to determine the difference between the populations in the bulge and disk of an S0 galaxy. This is also a way to measure the clumping and structure of the dust in dusty ellipticals, when the amount of dust is too small to characterize by visual appearance or color.

If the distance to a galaxy is known but is too great to identify and remove globular clusters, this technique can tell us about the specific frequency of globular clusters, since their variance is expected to dominate that of the luminosity fluctuations. For example, it should be possible to determine the frequency of globular clusters in the Coma cD galaxy N4874 and the Coma giant elliptical galaxy N4889. The *Hubble Space Telescope* will allow us to extend this method to greater distances by locating the point sources in high-signal-to-noise images taken from the ground. Once the (unseen) point sources are removed, the remaining variance should come from the luminosity fluctuations.

This paper has only begun to investigate what can be done by looking at the luminosity fluctuations in galaxies. Work is in progress to try to characterize $\bar{L}$ better, and to understand how well it can be determined from distance-independent measures of a galaxy's population, such as galaxy colors, line strengths, or detailed spectra. Additional observations are being made of galaxies throughout the Virgo supercluster, and in particular of the elliptical galaxies within the core of the Virgo cluster, since they are a diverse sample at the same distance. It is not clear at this point whether this technique will live up to its promise, but it certainly promises a great deal.

We believe that this is the first quantitative measurement of luminosity fluctuations, but the relationship between the resolution of stars, the resulting fluctuations, and the distance of the galaxy has occurred to many people before us. We are aware that William Baum has thought a great deal about this topic, particularly insofar as the resolution of *HST* will improve matters. This idea also was related to one of us (D.P.S.) by Peter Young, almost a decade ago. In many respects, this technique has only been waiting for the superb qualities of modern CCDs to come to fruition. We would like to thank Jim Gunn and Jim Westphal for their generous support with the instrumentation, and Ed Danielson and Dave Jewitt for aid in the M32 data acquisition. We are very grateful to Gerry Luppino and George Ricker for their CCD camera and for providing the observing time for N3379. Alar Toomre and Ed Bertschinger read many drafts of the manuscript and gave us invaluable comments and suggestions. This work was partially funded by NASA grants nos. NAS5-29225 and NSG7643, and an Exxon fellowship.

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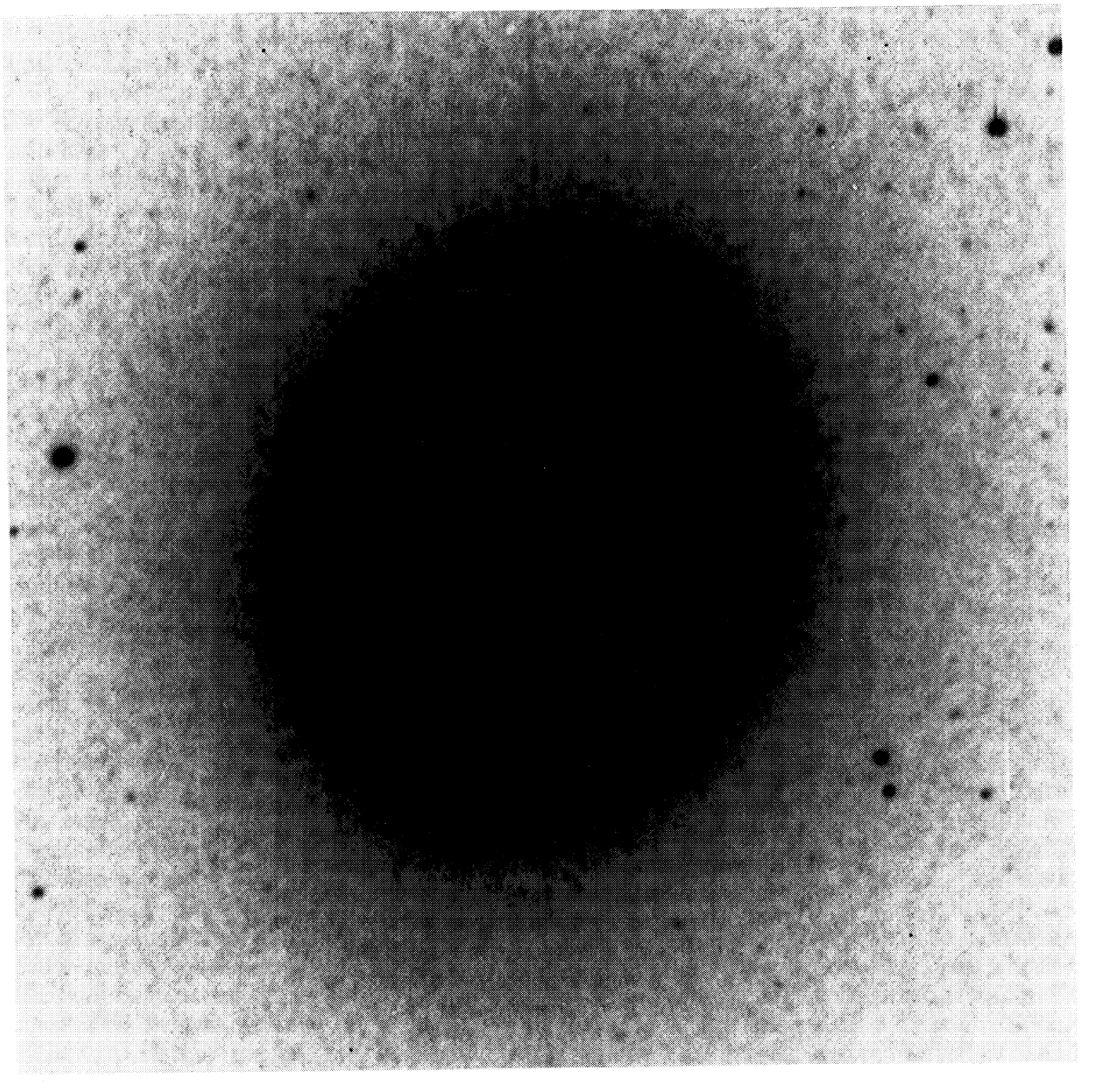

FIG. 1. The mottling due to fluctuations of the number of stars in a seeing disk is evident in this CCD picture of M32. This frame is the average of eight *r* exposures taken with the Hale telescope. The integration time for each exposure was 8 s, and the seeing was 1.3″ (FWHM). North is up, east to the left, and the picture is 207″ on a side.

J. Tonry and D. P. Schneider (see page 807)