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STATISTICAL ERROR ANALYSIS IN CCD TIME-RESOLVED PHOTOMETRY WITH APPLICATIONS TO VARIABLE STARS AND QUASARS

STEVE B. HOWELL[a)]
ST Systems Corporation, Code 681, NASA/Goddard Space Flight Center, Greenbelt, Maryland 20771

KENNETH J. MITCHELL[a),b)]
Laboratory for Astronomy and Solar Physics, Code 681, NASA/Goddard Space Flight Center, Greenbelt, Maryland 20771

ARCHIBALD WARNOCK III
ST Systems Corporation, Code 681, NASA/Goddard Space Flight Center, Greenbelt, Maryland 20771
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ABSTRACT

Differential photometric time series obtained from CCD frames are tested for intrinsic variability using a newly developed analysis of variance technique. In general, the objects used for differential photometry will not all be of equal magnitude, so the techniques derived here explicitly correct for differences in the measured variances due to photon statistics. Other random-noise terms are also considered. The technique tests for the presence of intrinsic variability without regard to its random or periodic nature. It is then applied to observations of the variable stars ZZ Ceti and US 943 and the active extragalactic objects OQ 530, US 211, US 844, LB 9743, and OJ 287.

I. INTRODUCTION

Accurate time-resolved photometry of faint ($V > 15$) astronomical sources is difficult to obtain with a conventional photoelectric photometer, traditionally requiring substantial blocks of time on a large (2–4 m class) telescope. The constraints imposed by telescope scheduling severely limit the amount of observing time available for such projects. Until recently, the alternative, at least for faint objects, has usually been to use photographic plates for which $\sigma \sim 0.1$ mag and the sampling time can seldom be less than 15 min.

Howell and Jacoby (1986) presented a method by which very accurate (<0.01 mag), relatively high-time-resolution differential photometry can be obtained on faint sources even with a moderate size (1 m or smaller) telescope. The method consists of obtaining images of the program object and at least two comparison stars in a single CCD frame, then performing differential photometry with the fluxes or the derived magnitudes. The three objects are traditionally called V (the program object), and C and K (the comparison stars). The calculated instrumental magnitudes are used to form the usual differences V − C and C − K. These differences, in general, will not be near zero.

In differential photometry, the C − K measures are typically used for three purposes. First, the C − K data can indicate trends and/or variability in the comparison stars; second, they serve to measure the intrinsic accuracy of the instrumental system; and third, they provide a comparison to determine if the program object V is, in fact, variable. Problems arise with conventional methods on nonphotometric nights because the sky conditions during observations of objects C and K may be different from those during observations of object V. The method of Howell and Jacoby (1986) improves on this situation by using a single CCD frame to generate simultaneous measures of C and K with each measure of V.

Because V, C, and K are all observed within one frame, CCD photometry differs from conventional photoelectric photometry in one important way. In conventional photometry, the integration times on each individual object may be scaled to yield the same S/N for all objects. On the other hand, the integration time for all of the objects in a single CCD frame is the same. As a result, each object in the CCD frame will have a different S/N ratio derivable from its magnitude. This difference makes it necessary to find a scale factor to correct for the different S/N ratios for the objects in the CCD frame *before* comparing V − C with C − K.

As part of a program to test CCD time-resolved photometry on astrophysically interesting objects, we obtained new time-ordered observations of one cataclysmic variable and five active extragalactic objects. All six have published histories of optical variability. We use the statistical analysis developed below to test for short-timescale variability in sequences of CCD frames of the objects. Other observational programs that have used this method can be found in Howell *et al.* (1987a,b) and Howell and Szkody (1987).

In this paper, we quantify the comparison of the V − C measures to those of C − K through statistical techniques that accurately estimate the expected variances. This allows testing for the presence of intrinsic variability over and above that expected from system noise and photon statistics alone. The test given here is not designed to determine the nature of the variations in a set of data. Rather, we seek merely to detect objectively the presence or absence of variability at some preassigned confidence level. The user then may apply other tests to determine the nature of the variability.

A derivation of the new statistical tests is given in Sec. II. Section III describes the observations and data-reduction techniques used, and Sec. IV gives the results of the variability analysis for each of the six objects observed. Section V contains a summary.

II. STATISTICAL ANALYSIS

a) Statistical Formalism

In this section, we formalize the use of the C − K observations in testing for intrinsic variability in the V − C data.

[a)] Visiting Astronomer, Kitt Peak National Observatory, National Optical Astronomy Observatories, operated by AURA, Inc., under contract with the National Science Foundation.

[b)] National Research Council—NASA Research Associate.

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Following usual statistical convention, we use σ_x^2 to denote the expected variance of the quantity X. In practice, we often do not know the true value of σ_x^2. We use s_x^2 to denote the estimate of σ_x^2, calculated from the observations X_i in the usual way,

$$s_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \overline{X})^2. \tag{1}$$

The basis of our statistical test is an analysis of the variance of V − C, denoted $\sigma_{\rm V-C}^2$. If object V is intrinsically variable, while C and K are not, then $\sigma_{\rm V-C}^2$ has two components. We shall write

$$\sigma_{\rm V-C}^2 = \sigma_{\rm V-C}^2({\rm VAR}) + \sigma_{\rm V-C}^2({\rm INST}), \tag{2}$$

where $\sigma_{\rm V-C}^2({\rm VAR})$ is the variance of V − C arising from the variability of V and $\sigma_{\rm V-C}^2({\rm INST})$ is the variance from all other sources (primarily instrumental). There is an expression similar to Eq. (2) for $\sigma_{\rm C-K}^2$, but by assumption, $\sigma_{\rm C-K}^2({\rm VAR}) = 0$, so $\sigma_{\rm C-K}^2 = \sigma_{\rm C-K}^2({\rm INST})$. There is always the possibility that some undetected variability exists in one or both of the comparison stars. This will not negate the validity of the method described below, but only leads to a more conservative result. The statistical test presented here requires that C and K do not vary significantly and it is the responsibility of the observer to verify that this assumption is valid. One way of doing this would be to apply the variability test to C and K individually, using additional stars in the CCD frame.

To test whether $\sigma_{\rm V-C}^2({\rm VAR}) = 0$, we must first estimate $\sigma_{\rm V-C}^2({\rm INST})$. We do this by deriving an expression that gives $\sigma_{\rm V-C}^2({\rm INST})$ in terms of $\sigma_{\rm C-K}^2$. This expression will explicitly correct for the photon-noise contribution arising from the differences in magnitudes between the three objects V, C, and K. For this purpose, we define a scale factor Γ^2 as

$$\Gamma^2 = \frac{\sigma_{\rm V-C}^2({\rm INST})}{\sigma_{\rm C-K}^2}. \tag{3}$$

We will calculate Γ^2 by estimating $\sigma_{\rm V-C}^2({\rm INST})$ and $\sigma_{\rm C-K}^2$ from the known properties of the CCD. We then can use Γ^2 to scale the measured variance $s_{\rm C-K}^2$, into $s_{\rm V-C}^2({\rm INST})$, the estimated variance of V − C in the absence of variability, using

$$s_\Gamma^2 = s_{\rm V-C}^2({\rm INST}) = \Gamma^2 s_{\rm C-K}^2. \tag{4}$$

(We will use s_Γ^2 in place of $s_{\rm V-C}^2$ whenever we want to stress our reliance on the scale factor Γ^2 in the calculations.) As Eq. (4) shows, any error present in $s_{\rm C-K}^2$ will be scaled by Γ^2 into s_Γ^2. Therefore, methods to reduce the uncertainty in $s_{\rm C-K}^2$ should be used, such as maximizing the number of data points and using integration times that will lead to good signal-to-noise in both C and K, as well as V.

This formalism has the advantage that all random-noise components in the C − K light curve (even those not explicitly modeled in the CCD-based error equation below) will be transferred, through Γ^2, to the estimated variance of V − C.

The notation of this section is summarized in Table I.

TABLE I. Notation.

$\sigma_{\rm C-K}^2$	—the variance of C − K predicted by the CCD-based error equation and the median C and K measurements.
	—estimated by $s_{\rm C-K}^2$ using the C − K observations.
$\sigma_{\rm V-C}^2 = \sigma_{\rm V-C}^2({\rm VAR}) + \sigma_{\rm V-C}^2({\rm INST})$	
	—estimated by $s_{\rm V-C}^2$ using the V − C observations.
$\sigma_{\rm V-C}^2({\rm VAR})$	—the variance of V − C due to the intrinsic variability of object V.
$\sigma_{\rm V-C}^2({\rm INST})$	—the variance of V − C predicted by the CCD-based error equation and the median V and C measurements.
	—estimated by $\Gamma^2 s_{\rm C-K}^2$

b) Assumptions and Notes

We assume that C and K have been chosen so that they have no intrinsic variability and that the C − K data include all system-dependent errors, and thus indicate the true accuracy obtained in a given run. We use these data to set limits on the accuracy of our measurements of V − C so that our results will not be biased by any intrinsic variability of the object V.

The measured sky value will be the same for all stars on a single CCD frame, and we use a single median sky value for a given sequence of frames. We find that this adequately represents the sky contribution to the photometric noise during a given sequence of CCD frames, even when the sky background changes during the sequence (e.g., moonrise). We use the median magnitudes for all of the program objects (V, C, and K) in the final form of our equations as well. The use of median values, rather than means, minimizes the impact on the equations of large excursions in the data, as might occur during a flare or eclipse in one of the program objects, or from an increase in extinction due to a passing cloud. Section II*d* discusses these effects in more detail.

We further assume that the photon statistics and any CCD-based noise variations are independent for the three objects. The separate V, C, and K data do have dependent noise terms due to extinction and a common sky background on a single CCD frame, but taking magnitude differences (or, equivalently, flux ratios) produces two independent sets of data.

Note that, even in the absence of intrinsic variability, one would expect variations due to photon (Poisson) statistics. This implies that even if none of the program objects is intrinsically variable, the ratio $\sigma_{\rm V-C}^2/\sigma_{\rm C-K}^2$ will *not*, in general, equal one. However, once $\sigma_{\rm C-K}^2$ is scaled by Γ^2, the corrected ratio will equal 1 exactly when none of the objects vary. This may be seen in detail from Eqs. (2) and (4), from which it follows that

$$\frac{\sigma_{\rm V-C}^2}{\Gamma^2\sigma_{\rm C-K}^2} = \frac{\sigma_{\rm V-C}^2({\rm INST}) + \sigma_{\rm V-C}^2({\rm VAR})}{\sigma_{\rm V-C}^2({\rm INST})}. \tag{5}$$

This ratio is precisely 1 if and only if $\sigma_{\rm V-C}^2({\rm VAR}) = 0$.

c) The Derivation of Γ^2

The total number of photons (or electrons) detected only from object X is denoted N_x, and is calculated by applying standard two-dimensional aperture-photometry techniques to the CCD frames. The integration time per frame, in seconds, is denoted t, and the electron flux A_x is given by $A = N_x/t$.

For a random variable w, which is a function of two independent random variables u and v (i.e., $w = f(u,v)$), we have

$$\sigma_w^2 = \sigma_u^2\left(\frac{\partial w}{\partial u}\right)^2 + \sigma_v^2\left(\frac{\partial w}{\partial v}\right)^2. \tag{6}$$

We now let $w = A_C / A_K$. Substituting this expression into Eq. (6) gives

$$\sigma^2_{(A_C/A_K)} = \frac{A_K^2 \sigma^2_{A_C} + A_C^2 \sigma^2_{A_K}}{A_K^4} \, (e^-/\mathrm{s})^2. \tag{7}$$

For convenience, we wish to compare variances measured in magnitudes, and so must transform the variance in Eq. (7) from electron flux to magnitude.

For random variables w and u with $w = g(u)$, we have

$$\sigma^2_w = \sigma^2_u \left(\frac{\partial w}{\partial u}\right)^2, \tag{8}$$

and we find from Eq. (8) and the standard equation relating flux ratio to magnitude difference that

$$\sigma^2_{C-K} = \left(\frac{2.5 \log e}{A_C/A_K}\right)^2 \sigma^2_{(A_C/A_K)} \, (\mathrm{mag})^2. \tag{9}$$

Substituting Eq. (7) into Eq. (9) gives the expression for the variance of the magnitude difference C − K in terms of the variances of the electron fluxes,

$$\sigma^2_{C-K} = (2.5 \log e)^2 \left(\frac{A_K}{A_C}\right)^2 \left(\frac{A_K^2 \sigma^2_{A_C} + A_C^2 \sigma^2_{A_K}}{A_K^4}\right) (\mathrm{mag})^2, \tag{10}$$

and similarly for the variance of V − C.

In the absence of variability, we can use Eqs. (3) and (4) to express the relation between the expected variance of V − C and the expected variance of C − K, with σ_{V-C} and σ_{C-K} measured in magnitudes. Substituting Eq. (10) and its V − C counterpart into Eq. (3) yields

$$\Gamma^2 = \left(\frac{A_K}{A_V}\right)^2 \left(\frac{A_C^2 \sigma^2_{A_V} + A_V^2 \sigma^2_{A_C}}{A_K^2 \sigma^2_{A_C} + A_C^2 \sigma^2_{A_K}}\right), \tag{11}$$

where Γ^2 can now be interpreted as a scale factor accounting for the differences in magnitudes between the three stars.

For a given CCD observation, with $A = N/t$ (in e^-/s) and ignoring dark current, the expected variance in flux is given by (see, for example, Mortara and Fowler 1981)

$$\sigma^2_A = \frac{N_X + n_p (N_s + N_r^2)}{t^2} \, (e^-/\mathrm{s})^2, \tag{12}$$

where N_X = total (sky-subtracted) counts in object X(e^-/integration); N_s = sky photons (e^-/pixel/integration); N_r = read noise (rms e^-/pixel); t = integration time (s), n_p = number of pixels in the applied measuring aperture.

If we let $P = n_p (N_S + N_r^2)$ and use Eq. (12) in Eq. (11), we have

$$\Gamma^2 = \frac{\sigma^2_{V-C}(\mathrm{INST})}{\sigma^2_{C-K}} = \left(\frac{N_K}{N_V}\right)^2 \left[\frac{N_C^2 (N_V + P) + N_V^2 (N_C + P)}{N_K^2 (N_C + P) + N_C^2 (N_K + P)}\right], \tag{13}$$

which relates the two variances σ^2_{C-K} and σ^2_{V-C}(INST) by a factor depending only on the relative photon noise of the three stars plus a common noise term.

d) The Optimal Choice of C and K

The term P contains the median sky value derived from a given contiguous run of frames. We have found that even for sky values that change by a factor of 5–10 within a given run, the effect on Γ^2 is < 2%. The counts used for V, C, and K are also median values. One could actually calculate a Γ^2 value for each CCD frame, whereas the value calculated by Eq. (13) is a global value for a given sequence. Comparison of the Γ^2 values on a frame-by-frame basis, even when the variable showed 1 mag changes, resulted in less than a 10% difference between the individual and global Γ^2 values.

Since Γ^2 is simply a scaling factor, any error in s^2_{C-K} will be scaled into an error in s^2_{V-C}(INST). We have already discussed the fact that one wants to make s^2_{C-K} as accurate as possible. We now discuss limits on Γ^2. When $\Gamma^2 = 1.0$, the difference between s^2_Γ and s^2_{C-K} will be a minimum. Therefore, we examine some ways in which Γ^2 can be near one. Howell and Jacoby (1986) always picked V, C, and K with similar magnitudes simply from photon-statistic considerations. We see from Eq. (13) that this situation does indeed lead to $\Gamma^2 = 1.0$. In fact, this is true as long as V = K. There are other ways to get Γ^2 near 1. For example, cases where $K \leqslant V \leqslant C$ can lead to $\Gamma^2 \approx 1$. In this case though, the errors in both the V − C and C − K light curves are dominated by the errors in the photometry of C. One is thus left with a less stringent test of variability. In general, it seems that the best compromise is to choose a K approximately equal in magnitude to V and to choose a C somewhat brighter than V and K.

A typical time series of observations on an object may consist of a few to many hundreds of CCD frames. These are reduced as described in Howell and Jacoby (1986), yielding instrumental magnitudes and, thus, magnitude differences. In Sec. IV, we use the measured s^2_{C-K}, along with Eqs. (13) and (4), to give us an estimate for σ^2_Γ, which can be used as an indicator of the photometric accuracy of the V − C light curve.

e) Statistical Testing

It is now possible to test the measured variance in V − C to see whether the program object really shows variability beyond that expected from photon noise and random error. Note again that, because of the effects of counting statistics on derived magnitudes of stars with unequal fluxes, $\sigma^2_{V-C}/\sigma^2_{C-K}$ may not be near 1, even if V is not a variable, while, in the absence of variability, the corrected ratio $\sigma^2_{V-C}/\Gamma^2\sigma^2_{C-K}$ should equal 1.

Formally, we test the hypothesis

$$H_0: \sigma^2_{V-C} \leqslant \Gamma^2 \sigma^2_{C-K} = \sigma^2_{V-C}(\mathrm{INST}) \tag{14}$$

(i.e., V is not variable) against the alternative

$$H_A: \sigma^2_{V-C} > \Gamma^2 \sigma^2_{C-K} = \sigma^2_{V-C}(\mathrm{INST}) \tag{15}$$

(i.e., V is variable) with an F test, appropriate for testing the ratio of two variances (Neter and Wasserman 1974). The test statistic for variability is

$$F_{\mathrm{test}} = \frac{s^2_{V-C}}{s^2_\Gamma} = \frac{s^2_{V-C}}{\Gamma^2 s^2_{C-K}}, \tag{16}$$

and we reject H_0 at the confidence level α if

$$F_{\mathrm{test}} > F(1 - \alpha; n_1 - 1, n_2 - 1), \tag{17}$$

where n_1 and n_2 are the number of degrees of freedom (number of observations) used to compute s^2_{V-C} and s^2_{C-K}, respectively. The tabular value $F(1 - \alpha; n_1 - 1, n_2 - 1)$ (see, for example, Neter and Wasserman 1974, Table A-4) is commonly referred to as the threshold (or critical) value, i.e., we have $100(1-\alpha)\%$ confidence in rejecting H_0 only when the test statistic exceeds the threshold value. In nonstatistical terms, the threshold value tells us how much greater than 1 the ratio $s^2_{V-C}/\Gamma^2 s^2_{C-K}$ must be in order to reject the hypothesis of no variability with $100(1-\alpha)\%$ confidence. The choice of α is made *a priori* to set the acceptable error level, keeping in mind that the sensitivity of the test must decrease as the confidence increases. In Sec. IV, we use the typical value $\alpha = 0.05$ to yield 95% confidence in the results.

III. OBSERVATIONS AND DATA REDUCTION

The observations presented here were made as part of a continuing program to test the method of CCD time-resolved photometry on various types of astronomical objects. We present new data on a faint cataclysmic variable and on five active extragalactic objects.

The observations were all obtained in March 1986 with the No. 1 0.9 m telescope at Kitt Peak National Observatory using the RCA1 CCD (1×1 pixel summed, gain $= 11.5\ e^-$/ADU, read noise = 79 electrons/pixel) and a Johnson V filter. Table II presents an observing log for the program objects. We observed standard stars several times each night in order to transform the observations from instrumental magnitudes to actual V magnitudes. No corrections for zenith-angle-dependent extinction were attempted. The observed standards included Landolt stars and standard stars in Kapteyn Selected Areas 28 and 57 kindly provided by A. Sandage (1982).

The individual calibrated observations of the program objects (Table II(a)) were reduced using standard 2-*D* digital aperture-photometry techniques, while the differential-time-series observations (Table II(b)) were obtained and reduced as described in Howell and Jacoby (1986). Examination of the raw counts in each series for the comparison objects C and K show variations of order 10% or less over the entire observing run. Thus, individual instrumental magnitude estimates should have errors of $\leqslant 0.1$ mag simply due to prevailing sky conditions. We adopt this value as our error in the calculated V_{pe}, V_{pg}, and B_{pg} magnitudes because it is slightly more conservative than the formal-prediction standard deviation. However, this error is not the same as s_Γ, the differential-time-series error.

In order to determine approximate V magnitudes from our CCD instrumental V magnitudes, we have used the observed standard stars to model color terms in the CCD data and to verify the errors in our derived V magnitudes. We can then determine the instrumental magnitude offset as a function of the observed $B-V$ color index. Figures 1 and 2 show the modeled color effects separately for standards with photoelectrically determined magnitudes and for those with photographically determined magnitudes. As noted in the captions for Figs. 1 and 2, the average V magnitudes are plotted and the standard deviations of the given average magnitudes are nearly all smaller than the size of the plotted symbol.

Figure 1 shows the difference $(V_{CCD} - V_{pe})$ plotted against $B-V$ for a number of standard stars obtained from three sources: Landolt (1973), Mount Wilson (Sandage 1982), and Purgathofer (1969). We find that, over the range of $0.4 < B-V < 1.4$, no color terms are present and the data are best fit by a linear relation of the form $V_{pe} = V_{CCD}$

TABLE II(a). Single photometric observations.

UT Date	Object	Integration Time (seconds)	UT Mid-Exposure (approximate)	V_{pg}[1]
20 March 1986	US 943	1200	3:50	19.4[2]
		1200	8:40	19.4[2]
	LB 9743	300	9:20	17.3
		300	11:55	17.3
21 March 1986	OJ 287	120	3:00	15.2
		120	7:45	15.4
22 March 1986	OJ 287	120	2:46	15.3
		60	2:52	15.3
		60	7:15	15.3
	US 943	1200	3:20	19.1[2]
		1200	8:00	18.7[2]
	LB 9743	150	8:21	17.4
		150	12:27	17.5
	US 211	1200	12:04	19.4

Notes:

1) V_{pg} from photographic V relation, see text.
2) magnitude given is V_{pe}, see text.

TABLE II(b). Time-series observations.

UT Date	Object	Integration Time (sec)	No. of Observations	Total Time Observed (min)	UT Start of series (mid-exp)	Range in V_{pg}[1]
20 March 1986	OJ 287	120	100	200	4:21:32	15.1-15.1
	US 211	1200	6	120	9:59:25	19.2-19.4
21 March 1986	US 943	1200	11	220	3:54:28	18.8-19.8[2]
	LB 9743	150	68	170	8:36:27	17.3-17.4
22 March 1986	US 844	900	13	180	4:09:36	18.9-19.1
	OQ 530	120	84	168	8:43:39	15.6-15.7

Notes:
1) V_{pg} from photographic V relation, see text.
2) magnitudes are V_{pe}, see text.

-3.38 with a root-mean-square error of 0.081 mag. We use all three standard-star data sets here. The larger number of stars for our fit increases the power of our modeling.

Figure 2 shows a similar plot, using standard stars with photographically determined V magnitudes (Sandage 1982). These stars cover a larger range of $B-V$ (approximately 0.0 to $+1.6$) and were fit by the relation $V_{pg} = V_{CCD} - 0.77(B-V+0.2)^{1/2} - 2.54$. The fitted relation has a root-mean-square error of 0.087 mag.

For consistency in the analysis that follows, we have used the photographic relation for calculating the calibrated V magnitudes of all the extragalactic objects, since a few have very blue colors. The photoelectric relation was used to derive V magnitudes for US 943.

IV. RESULTS

We now apply the statistical methods of Sec. II to the newly obtained differential time series in order to see how well the method estimates light-curve variances and to test for intrinsic variability. All of the objects observed have histories of optical variability and/or variability at other wavelengths. The extragalactic objects are good candidates for short-timescale variability (or flickering) that could origi-

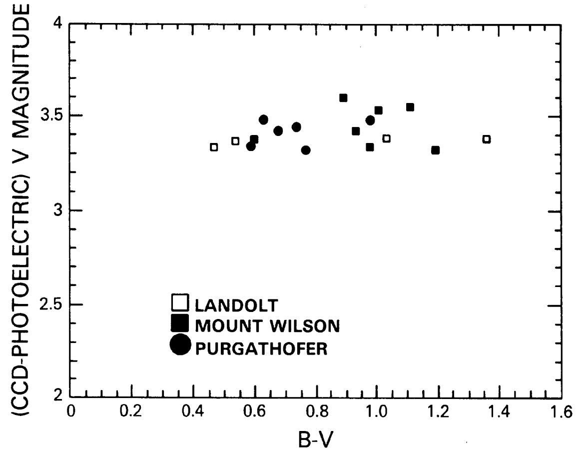

FIG. 1. Photoelectric V magnitude relation. Shown is the instrumental CCD V magnitude minus the published photoelectric V magnitude plotted against $B-V$ for a number of photometric standard stars from the references indicated. Except for the two bluest Landolt stars, which are single observations, the plotted symbols represent average values for each of the standards; Landolt = 3, Mount Wilson = 4 or 5, and Purgathofer = 5 measurements. The Mount Wilson point at $B-V=0.6$ is plotted on top of a Landolt measurement. All of the variances of the means are within the plotted symbols except for the Purgathofer points at $B-V=0.68$ and 0.74, which have $\sigma=0.05$ and 0.12, respectively. See the text for details on the fitted relation.

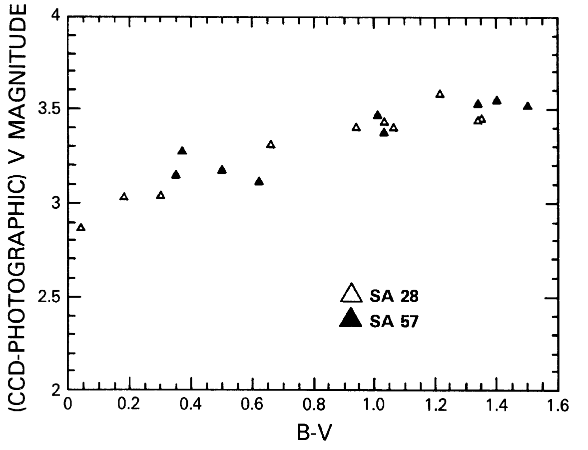

FIG. 2. Photographic V magnitude relation. Shown is the instrumental CCD V magnitude minus the photographic V magnitude plotted against $B-V$ for a number of photometric standard stars from Kapteyn Selected Areas 28 and 57. The plotted symbols all represent average values for each standard; SA 28 = 4 and SA 57 = 5 measurements. All the variances of the means are within the plotted symbol except for the SA 28 points at $B-V=0.30$, 0.18, and 0.04, which have $\sigma=0.10$, 0.12, and 0.15, respectively. See the text for details on the fitted relation.

nate in active compact or relativistically moving components. As such, it would be completely inappropriate to try to detect variability by applying a period-finding algorithm, such as an FFT. Two of the objects are known optically violent variable BL Lacertae objects. To further verify the efficacy of the statistical method in detecting variability, we also re-evaluate previously published time-resolved CCD data on the star ZZ Ceti (Howell and Jacoby 1986).

Table III contains all of the relevant quantities derived from the time-series observations. Each column is numbered and has a note to more fully explain its meaning. More specific notes for individual objects are listed in column 14. We discuss our results for each object separately below. All tests in this section have been performed at the 95% confidence level.

a) ZZ Ceti

Howell and Jacoby (1986) have presented data on the vibrating white dwarf ZZ Ceti. This star shows two closely spaced periods of 213 and 274 s and an amplitude of variation of 0.01 mag. Their conclusion was that intrinsic variations were actually observed (see their Fig. 4). In order to verify this claim and to test our statistical method described in Sec. II, we apply it to their original data set. We see that, at the 95% confidence level, variability in their observations is confirmed. This confirmation can and does only detect the presence or absence of variability and is not a detector of periodicity. Howell and Jacoby used a period-finding algorithm to identify the periods present in the ZZ Ceti data.

b) US 943

US 943 is a $B \simeq 19$ mag stellar object with colors at the selection epoch similar to the halo F/G subdwarfs. It was found to be optically variable with large variability amplitudes by Usher *et al.* (1982). Our observations, presented in Fig. 3, consist of 11 20 min integrations. The V − C light curve shows variations on timescales of 20 to 40 min with amplitudes of up to 1 mag. The expected rms noise in the V − C light curve is $s_\Gamma = 0.05$ mag, and the statistical test confirms the presence of significant variability. These observations have already been analyzed in terms of classifying US 943 as a halo cataclysmic variable (Howell *et al.* 1987a,b).

TABLE III. Photometric and statistical properties.

Object	V	V-C	C-K	n	t	n_p	sky	s^2_{V-C}	s^2_{C-K}	Γ^2	F_T	Var	Notes
1	2	3	4	5	6	7	8	9	10	11	12	13	14
ZZ Ceti	14.9	-0.3	+0.1	27	80	25	2160	9.0(-5)	1.4(-5)	0.91	7.10	y	a,b,c
US 943	19.1	-0.7	+0.6	11	1200	23	2490	0.1162	0.0031	0.96	39.1	y	d,e,f
OQ 530	15.6	+1.0	-1.4	85	120	78	100	1.4(-4)	2.2(-4)	0.54	1.17	n	d,g
US 211	19.3	+0.1	+0.6	6	1200	44	295	0.0032	0.0019	1.62	1.04	n	d
US 844	18.9	+2.5	-1.6	13	900	50	2947	0.0039	6.9(-4)	4.89	1.16	n	d
LB 9743	17.3	+0.4	-1.3	68	150	40	80	7.7(-4)	0.0019	0.60	0.67	n	d
OJ 287	15.1	+0.4	-0.3	100	120	14	600	1.4(-4)	7.9(-5)	1.09	1.63	y	d

Notes to columns:

1) object name
2) median calibrated V magnitude for the time series
3) median magnitude offset between V and C for the time series
4) median magnitude offset between C and K for the time series
5) number of observations in the time series
6) integration time per CCD frame, in seconds
7) number of pixels in the software star aperture
8) median sky level for the time series, in ADU/pixel/integration
9) measured variance for V-C
10) measured variance for C-K
11) statistical correction factor, Γ^2
12) test statistic F_{TEST} for the F-ratio test, see eq. (16)
13) variability result at the 95% confidence level ($\alpha = 0.05$)
14) Additional notes:
    a) see Howell and Jacoby, 1986
    b) TI2 CCD, 2x2 pixel summed, gain = 4.3 e^-/ADU, read noise = 18 e^-
    c) instrumental magnitudes only
    d) RCA1 CCD, 1x1 pixel summed, gain = 11.5 e^-/ADU, read noise = 79 e^-
    e) see Howell *et al.* 1987a, b
    f) photoelectric magnitudes
    g) see section IV.

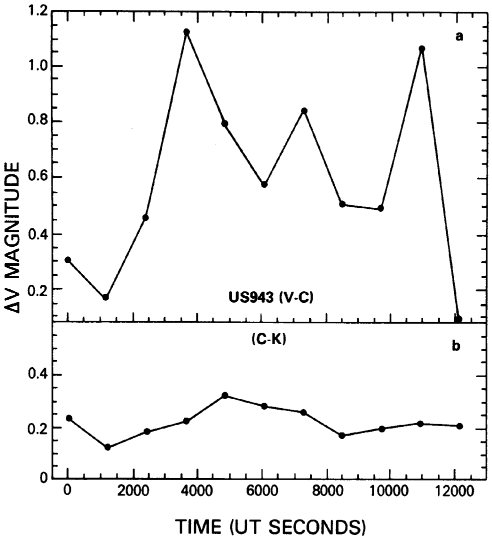

FIG. 3. Differential photometry for US 943. Part (a) shows the V − C differential light curve, while part (b) shows the C − K differential light curve. Each point represents the time of mid-exposure with the first point in each plot being that time given in Table II(b) for the start of the time series. An estimate of the photometric accuracy in the differential V data may be obtained by using s_Γ, which can be calculated from the values given in Table III.

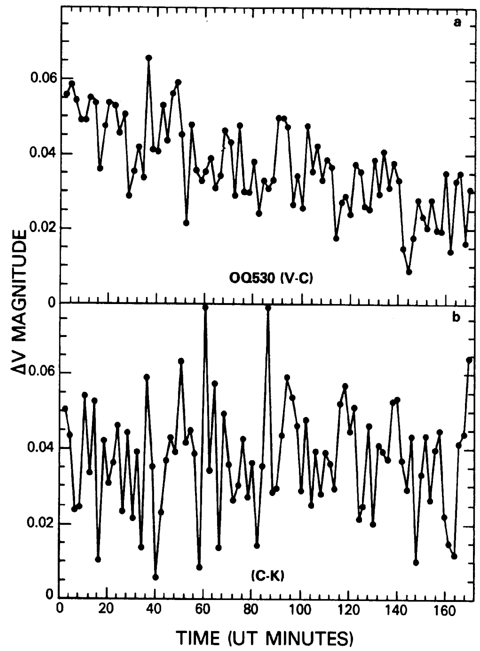

FIG. 4. Differential photometry for OQ 530. See Fig. 3 caption for a complete description.

c) OQ 530

This BL Lacertae object was first noted as a radio-source optical identification (Kühr 1977) exhibiting large optical linear polarization (Craine *et al.* 1978). Subsequent study of OQ 530 by Miller (1978) using the Harvard College Observatory plate archive showed it to be highly variable, with a total range of ∼4.8 mag. Miller's study also revealed short-timescale variability (on the order of days).

The statistical test of Sec. II rejects, at the 95% confidence level, the hypothesis of variability. Our sequence of 85 2 min observations is shown in Fig. 4. Notable in the V − C data is a steady dimming trend for OQ 530. We fit a linear trend model to these data, and an analysis-of-variance study reveals the trend to be significant at the >95% level, with a slope of $1.66\ (\pm 0.19) \times 10^{-4}$ mag per minute. This trend is not seen in the C − K data, and the C and K data alone provide no evidence for any long-term extinction trends. Thus, we feel that the observed trend is not caused by nongray extinction or problems on the CCD chip, and that it is intrinsic to the source. The test fails to detect significant variability at the 95% confidence level because the C − K data are noisier than V − C (the variability would have been significant had the confidence level been set at 85%).

The expected variance in the V − C data calculated from the C − K data is almost exactly that observed, including the trend. The regression analysis indicates that roughly half of the variance in the V − C light curve is due to the linear trend and the other half to random noise. Thus, after subtraction of the trend, we are left with an rms noise ($s_{V-C} = 0.008$ mag) in the V − C light curve that is smaller by a factor of $\sqrt{2}$ than that expected from pure photon statistics and Eq. (10). We stress that if the trend were eliminated by randomly rearranging the observations (this does not affect the overall variance), no variability would have been recognized.

By inverting the measured slope of the trend in the V − C light curve of OQ 530, we can produce a variability timescale that represents an inverse rate of change of the fractional flux coming through the V filter (Condon *et al.* 1979). We measure this timescale to be ∼4 days, which is comparable to the timescale of ∼2 days measured by Miller (1978) during one of the violent optical outbursts of OQ 530. Thus, we appear to be witnessing in the V − C data a "violent" decrease in flux by OQ 530. We further note that in the V − C light curve there is no significant structure on the timescale of minutes superimposed on the ∼4 day timescale.

The average calibrated V_{pg} magnitude measured here can be converted to an approximate $B_{pg} \simeq 16.0$ by assuming $(B-V) \simeq +0.4$ as is measured for OJ 287, another BL Lacertae object. This value is close to the faintest B magnitude ($\simeq 16.1$) ever observed for this object (Miller 1978) and, if the trend in Fig. 8 continued, it would take only another half day or so for OQ 530 to bypass this faint limit.

d) US 211

First cataloged as a $B = 18.9$ mag stellar object with halo F/G subdwarf colors (Usher 1981), US 211 was later found to vary optically with an extremum amplitude of $\Delta B \simeq 1$ mag (Usher *et al.* 1983b). Figure 5 shows our sequence of six 20 min observations. The rms variation in the V − C light curve is measured to be $s_{V-C} = 0.06$ mag. This value of s_{V-C} is approximately that expected due to noise, and thus provides no evidence of significant short-timescale variability in the differential photometry.

We note here that our ability to detect low-level variability is severely limited because the light curve consists of only six points. The test, while still accurate at the 95% level, is less sensitive than one based on a larger number of points because the threshold statistic is large when there are few observations (see Eq. (17)).

Longer-term variability is indicated by our value of V_{pg}.

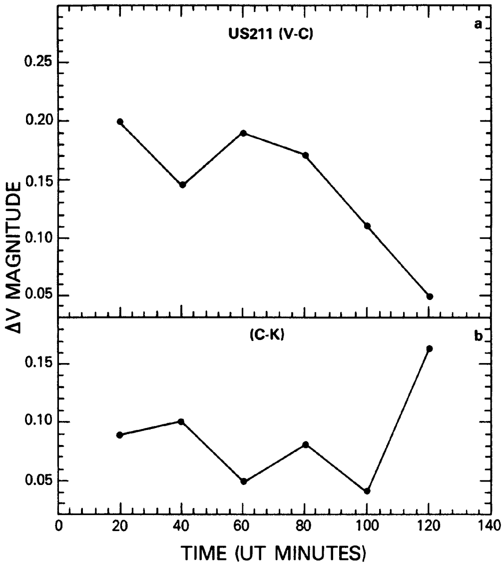

FIG. 5. Differential photometry for US 211. See Fig. 3 caption for a complete description.

This value can be converted to $B_{pg} = 19.6 \pm 0.2$ mag through the use of $(B-V)$ colors estimated at the time US 211 was cataloged. Combined with the cataloged $B_{pg} = 18.9 \pm 0.1$, our new value produces an $\sim 95\%$ detection of variability with $\Delta B \simeq 0.7$ mag. The value of B_{pg} we measure here falls within the extremum range of magnitudes observed by Usher *et al.* (1983b).

e) US 844

This object was first cataloged as a moderately ultraviolet-excess stellar object with $B = 18.7$ (Usher *et al.* 1982), and subsequently identified as a variable quasar with $\Delta B \simeq 0.9$ mag (Usher *et al.* 1983a,b). Figure 6 shows our sequence of 13 15 min integrations. An rms of $s_{V-C} = 0.06$ mag is measured for the V − C light curve, and provides no evidence for short-timescale variability. Our value of $V_{pg} = 18.9 \pm 0.1$ transforms into an estimated $B_{pg} = 19.3 \pm 0.2$ through use of an estimated $(B-V) \simeq +0.4$ mag (Usher, private communication). This B_{pg} is a new faint extremum for US 844 (e.g., Usher *et al.* 1983a).

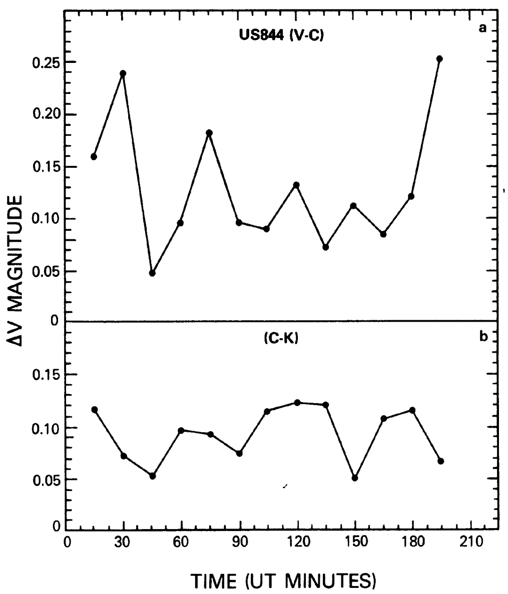

FIG. 6. Differential photometry for US 844. See Fig. 3 caption for a complete description.

f) LB 9743

This quasar was first cataloged by Luyten *et al.* (1967), with spectroscopic identification by Schmidt (1974). Optical variability, significant at the $\sim 3\sigma$ level with an extremum amplitude of $\Delta B \simeq 0.3$ mag, was found by Usher and Mitchell (1978). Variability on 200 s timescales at x-ray wavelengths has been claimed by Matilsky *et al.* (1982). Our sequence of 68 2.5 minute integrations is shown in Fig. 7. The rms variation of $s_{V-C} = 0.03$ mag measured for the V − C light curve is consistent with the expected noise; thus, no short-timescale variability is detected. Our average magnitude of $V_{pg} = 17.3 \pm 0.1$ mag, converted to $B_{pg} = 17.1 \pm 0.1$ mag through a value of $(B-V) \simeq -0.2$ mag (Luyten *et al.* 1967), is 0.7 mag fainter than the average $B_{pg} = 16.39 \pm 0.05$ measured by Usher and Mitchell (1978). The difference is significant at the $\sim 6\sigma$ level.

g) OJ 287

OJ 287 is a well-known active BL Lacertae object whose history and light curves at optical, infrared, and radio wavelengths are reviewed by Usher (1979). Evidence for short-timescale variability is also reviewed there, with the existence of day-to-day variations being especially well established within a number of references. Figure 8 shows our sequence of one hundred 2 min integrations. The statistical method of Sec. II indicates that, at the 95% confidence level, the measured variance of the data in the V − C light

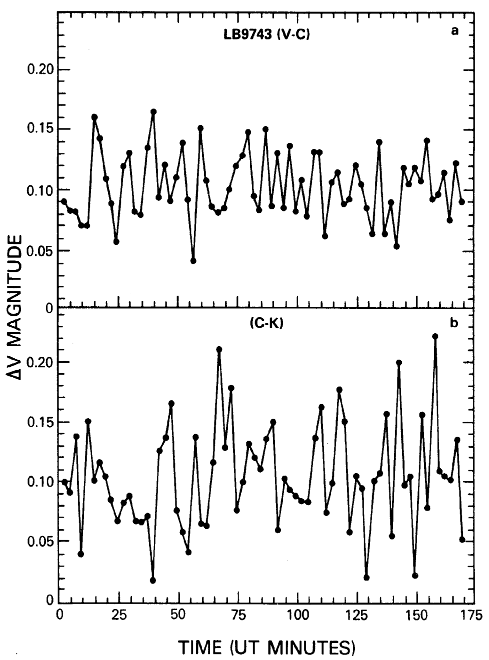

FIG. 7. Differential photometry for LB 9743. See Fig. 3 caption for a complete description.

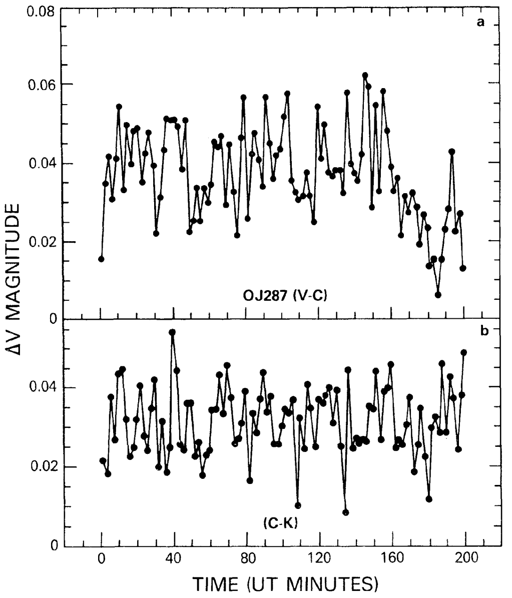

FIG. 8. Differential photometry for OJ 287. See Fig. 3 caption for a complete description.

curve is significantly greater than that expected from random noise alone.

Towards the end of the sequence of observations, for about 30 min, OJ 287 became ~ 0.02 mag fainter than in the initial 160 min. This feature does not appear in the C − K data, and the variance for the first 160 min of observations ($s_{V-C} = 0.010$ mag) is consistent with the expected noise; thus, we feel that the variability feature is real. The feature runs into the end of our data and we are unable to determine if this is a long-term change of flux level or a shorter-timescale dimming with a return to the initial flux level after our observations ended.

In any case, the change of 0.02 mag took no more than 32 min to occur. This timescale is remarkably similar to those found in 4C 29.45 by Grauer (1984), who obtained more complete observational coverage of several short-timescale changes in the flux of that source. Our observations of OJ 287 confirm that there are interesting short-timescale, small-amplitude optical variations that occur in at least some of the very active extragalactic objects.

The calibrated V_{pg} magnitudes that we measure for OJ 287 on each of the three nights show no evidence for larger-amplitude ($\Delta V_{pg} > 0.3$) variability. Through the use of $(B-V) = +0.4$ (Kinman 1976), we obtain $B_{pg} = 15.6 \pm 0.1$, which is closer to the nominal quiescent levels of $B \simeq 16$–16.5 than to the levels of $B \simeq 12.5$–14 reached by OJ 287 during outburst (Usher 1979).

V. SUMMARY

We have shown that the method of Howell and Jacoby (1986) can be used with the above statistical treatment to perform tests for variability in accurate time-resolved CCD photometry on astronomical sources. In fact, a treatment such as that given here may be applied to any two-dimensional detector performing differential observations, provided that one can obtain the appropriate analog to Eq. (12). Since our method uses a CCD detector in its usual direct image mode, no modifications to the hardware are needed, and, if desired, one can write software for "at the telescope" reductions. These reductions should be done in real time, so they can provide the observer with immediate information about the current state of the suspected variable. This type of real-time analysis would have been useful, for example, during our observations of OJ 287.

Stover (1986) has built a CCD system specifically designed for high-speed photometry which incorporates many real-time features, including the use of a weighting function to allow better sampling of the stellar profile. This is the desirable alternative but is not possible with the CCD systems currently available at Kitt Peak or other observatories without significant system modifications. In the absence of such a specialized instrument, it is still useful to calculate Γ^2 for each frame as the series progresses, and to apply a weighting scheme to the stellar profile.

For the present, the method of repeated exposures holds great promise in investigating the short-timescale behavior of faint objects. CCD camera systems are now widely available for small telescopes, and so are accessible to a large number of astronomers desiring accurate differential photometry, especially for faint sources. As new CCD imagers are built, we hope that their direct-imaging capabilities will include provisions for high-speed photometric observations as well.

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