

The Ups and Downs of the Hubble Constant

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Abstract

A brief history of the determination of the Hubble constant H_0 is given. Early attempts following Lemaître (1927) gave much too high values due to errors of the magnitude scale, Malmquist bias and calibration problems. By 1962 most authors agreed that $75 \lesssim H_0 \lesssim 130$. After 1975 a dichotomy arose with values near 100 and others around 55. The former came from apparent-magnitude-limited samples and were affected by Malmquist bias. New distance indicators were introduced; they were sometimes claimed to yield high values of H_0 , but the most recent data lead to H_0 in the 60's, yet with remaining difficulties as to the zero-point of the respective distance indicators. SNe Ia with their large range and very small luminosity dispersion (avoiding Malmquist bias) offer a unique opportunity to determine the large-scale value of H_0 . Their maximum luminosity can be well calibrated from 10 SNe Ia in local parent galaxies whose Cepheids have been observed with HST. An unforeseen difficulty - affecting all Cepheid distances - is that their P-L relation varies from galaxy to galaxy, presumably in function of metallicity. A proposed solution is summarized here. The conclusion is that $H_0 = 63.2 \pm 1.3$ (random) ± 5.3 (systematic) on all scales. The expansion age becomes then (with $\Omega_m = 0.3, \Omega_\Lambda = 0.7$) 15.1 Gyr.

1 Introduction

The present value of the Hubble parameter is generally called “Hubble Constant” (H_0). The *present* value requires minimum look-back-times; it is therefore to be determined at the smallest feasible distances and is adequately defined by

$$H_0 = \frac{v}{r} [\text{km s}^{-1} \text{Mpc}^{-1}], \quad (1)$$

where $v = cz$, $z = \Delta\lambda/\lambda_0$, and r = distance in Mpc. As long as $z \ll 1$, it is indicated to interpret cz as a recession velocity because the observer measures the *sum* of the space expansion term $z_{\text{cosmic}} = \mathcal{R}_0/\mathcal{R}_{\text{emission}} - 1$ (\mathcal{R} being the scale factor) and z_{pec} caused by the density fluctuation-induced peculiar motions. At small z_{cosmic} and in high-density regions z_{pec} is not negligible. It is therefore mandatory to measure $H_0(\text{cosmic})$ at distances where $z_{\text{cosmic}} \gg z_{\text{pec}}$ and outside of clusters. Any determination of H_0 must

therefore compromise between two conditions: the smallest possible galaxy distances r and a minimum influence of z_{pec} . The local Group is obviously useless for the determination of H_0 because it is probably gravitationally bound. The nearby Virgo cluster affects the local expansion field out to $\sim 2000 \text{ km s}^{-1}$ (see Section 6). At $v \sim 3000 \text{ km s}^{-1}$ the relative contribution of random velocities of field galaxies decreases to less than 10%, yet a volume of roughly similar radius has a bulk motion of 630 km s^{-1} with respect to the CMB. To be on the safe side it is therefore desirable to trace H_0 out to say $\sim 20\,000 \text{ km s}^{-1}$. The expansion rate at this distance is for all practical purposes still undistinguishable from its present value.

The first spectra of galaxies and the measurement of their radial velocities by Slipher (1914) and later by M. Humason and others was an epochal achievement. Today the observation of the redshifts needed for the calibration of H_0 is routine. The emphasis here lies therefore entirely on the determination of galaxy distances.

2 The First Galaxy Distances

While the question as to the nature of the “nebulae” was still wide open, Hertzprung (1914) applied the period-luminosity (P-L) relation of Cepheids, which he had calibrated with Galactic Cepheids, and found a distance modulus of SMC of $(m - M)_{\text{SMC}} = 20.3$ (115 kpc), roughly a factor 1.8 too large. According to the custom of the time he transformed the distance into a trigonometric parallax of $0.''0001$, losing a factor of 10 during the process. While transforming the parallax into light years he lost another factor of ten. Thus his published distance of 3000 light years buried his sensational result.

In the following year Shapley (1915) repeated Hertzprung’s measurement. For various reasons he now obtained a Cepheid distance of only $(m - M)_{\text{SMC}} = 16.1$ (17 kpc), which he slightly increased in 1918 and which he could take as a confirmation of his conviction that all “nebulae” were part of his very large Galactic system.

Lundmark (1920) was the first to recognize supernovae as a class distinct from novae. This explained the brightness of the “nova” 1885 in M31 and led him to a modulus of $(m - M)_{\text{M31}} = 21.3$ (180 kpc). Still much too low the value could not be accommodated within even the wildest size estimates of the Galaxy. But the result had no influence on the “Great Debate” (cf. Fernie 1970).

Öpik (1921, 1922) ingeniously used the rotation velocity of M31 to determine the mass-to-light ratio of the galaxy and he broke the distance degeneracy of this value by adopting a very reasonable mass-to-light ratio of the Solar neighborhood. He obtained a stunningly good value of $(m - M)_{\text{M31}} = 24.5$ (750 kpc), which he decreased in the following year by a factor of 1.7. Öpik’s papers remained unnoticed.

The discovery of several novae in “nebulae”, first by Ritchey (1917), stimulated the search for variability and led Hubble to the discovery of a Cepheid in

M 31 in 1923, – the first Cepheid beyond the Magellanic Clouds. At the meeting of the Association for the Advancement of Science in December 1924 he announced the discovery of several very faint Cepheids in M 31. They proved that many of the nebulae are actually “island universes”, but the proof was not yet generally accepted, because van Maanen’s (1923) claim of a detectable rotation of the spirals. Hubble published his Cepheid distance of M 31 only in 1929a, after he had published the Cepheids in NGC 6822 (1925) and M 33 (1926).

Hubble used his Cepheid distances to calibrate the brightest stars ($M_{pg} = -6^m3$) and the mean luminosity of “bright” galaxies ($M_{pg} = -15^m8$; either value being $4^m - 5^m$ too faint). In this way he extended his distance scale out to the Virgo cluster. In 1929b he plotted 31 of his distances against Slipher’s radial velocities. Not without remaining doubts, he concluded from the correlation of these two parameters that the Universe was expanding and that the expansion rate was $H_0 = 500$ – a value which he never decisively revised. His paper is generally considered to be the discovery of the expanding Universe, although Lemaitre (1927) and Robertson (1928) had anticipated the result and published expansion rates – using Hubble’s distances – corresponding to $H_0 = 627$ and 461 , respectively. de Sitter (1930) used 54 galaxy *diameters* and radial velocities out to the Coma cluster – again making extensive use of Hubble’s data – to derive $H_0 = 461$. Oort (1931), questioning Hubble’s bell-shaped galaxian luminosity function and increasing the luminosity of the really big galaxies, concluded that $H_0 \approx 290$. The result was important (yet hardly noticed) because it implied an expansion age of ~ 3.5 Gyr (For $q_0 = 0$) and removed the open contradiction with geological ages of the time.

For the next 20 years little was done on H_0 until Behr (1951) challenged Hubble’s value. He noticed the large luminosity scatter of Local Group galaxies and he argued via the Malmquist effect that Hubble’s *mean* luminosity was too faint by $\sim 1^m5$ if applied to more distant, magnitude-selected galaxies. (This is to my knowledge the first mentioning of Malmquist statistics in extragalactic work). Citing Baade (1944) he also corrected Hubble’s magnitudes by 0^m35 (at 18^m3). These were Behr’s two main reasons for deriving a value of $H_0 = 260$. He would have found an even smaller value had he known of Stebbins, Whitford, & Johnson’s (1950) pioneering photoelectric photometry which proved Hubble’s photometric scale error to be even larger.

The Malmquist (1920, 1922) bias of apparent-magnitude-limited samples as opposed to distance-limited samples (which are very hard to come by) was fully acknowledged by stellar astronomers since the 1920’s, but it has beset – if neglected – the extragalactic distance scale until quite recent times and led consistently to too high values of H_0 . The effect is illustrated in Fig. 1 and shows that in magnitude-limited samples the mean absolute magnitudes of “standard candles” with non-vanishing luminosity dispersion becomes brighter with increasing distance. – A smaller, but frequent overestimate of H_0 comes in case of several individual determinations by averaging over H_i , instead of over $\log H_i$.

In later years many ways have been proposed how to correct apparent-

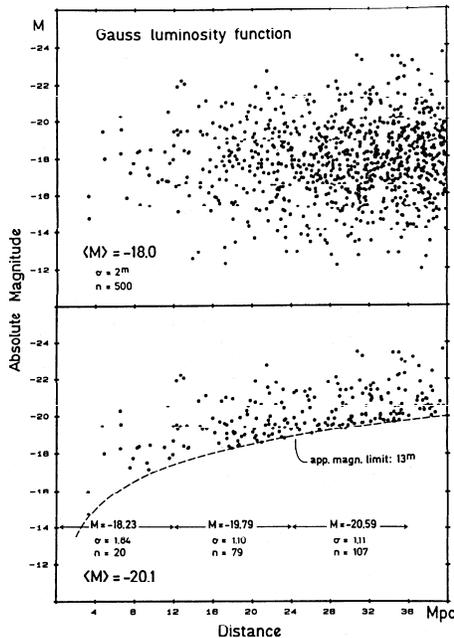


Figure 1: A Monte Carlo demonstration of the Malmquist Bias for 1000 “standard candles” of fixed mean luminosity (-18^m), non-zero luminosity dispersion ($\sigma = 2^m$) and $r < 40$ Mpc. Constant space density is assumed. Upper panel: The unbiased distribution in absolute magnitude of a *distance-limited* sample. Lower panel: The same sample, but cut by an *apparent-magnitude* limit (13^m). Note the increasing mean luminosity and decreasing magnitude dispersion in progressive distance intervals. (Only the magnitude dispersion of the entire sample in the lower panel happens to be close to the true dispersion in the upper panel). (By kindness of A. Spaenhauer).

magnitude-limited samples in general and of field galaxies in particular for Malmquist bias (e.g. Spaenhauer 1978; Tammann et al. 1979; Teerikorpi 1984, 1997; Bottinelli et al. 1986; Sandage 1996, 1999b, 2002; Theureau et al. 1997; Goodwin et al. 1997; Paturel et al. 1998; Ekholm et al. 1999; Butkevich et al. 2005, for a tutorial see Sandage et al. 1995). Also cluster samples are affected by “Teerikorpi Cluster Population Incompleteness Bias” (Teerikorpi 1987; Sandage et al. 1995). The hope that the inverse TF relation was bias-free has not substantiated (Teerikorpi et al. 1999). In all cases the correction for Malmquist bias requires large and fair samples.

Baade (1948) had described the determination of improved extragalactic distances as one of the major goals of the future 200” telescope. Contrary to Behr he stirred anything short of a sensation when he (1952) announced that work in M31 had shown, that either the zero-point of the Cepheids or of the RR Lyr stars must be in error. Since Sandage’s (published 1953) color-

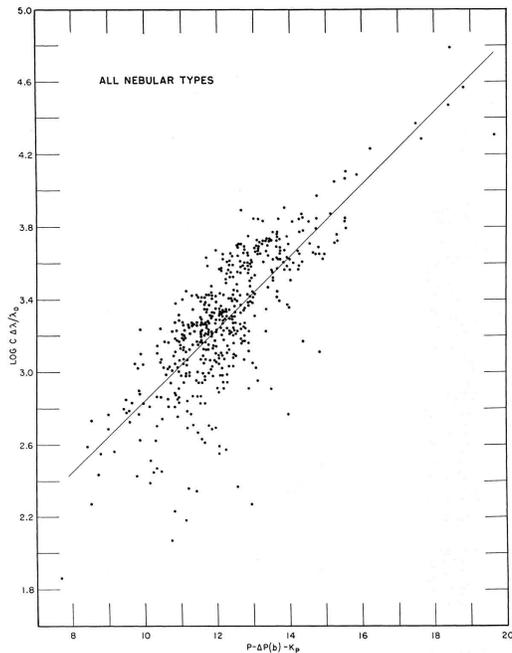


Figure 2: The Hubble diagram of the 474 field galaxies with redshifts known in 1956. The photographic magnitudes are corrected for Galactic absorption and the K-effect (due to redshift). The full line has slope 0.2 corresponding to linear expansion. A fit to the data gives a steeper slope, because the mean luminosity increases with distance due to Malmquist bias. (From Humason et al. 1956).

magnitude diagram of M3 had shown that the RR Lyr stars are correct, the Cepheid luminosities had to be increased, as Mineur (1945) had already suggested. Baade concluded that “previous estimates of extragalactic distances . . . were too small by as much as a factor of 2”, which led him to $H_0 \sim 250$. Accounting for the first four years of research with the 200" telescope, Sandage (1954), including also novae, summarized the evidence for H_0 and concluded $125 < H_0 < 276$.

In their fundamental paper Humason, Mayall, & Sandage (1956) estimated $H_0 = 180$ on two grounds: (1) They showed that what Hubble had considered as brightest stars of NGC 4321, a member of the Virgo cluster, were actually HII regions. The brightest stars set in only $\sim 2^m$ fainter. (2) The absolute magnitude of M 31, resulting from its apparent Cepheid modulus of $(m-M) = 24.25$ (Baade & Swope 1954), could be used by the authors to calibrate the *upper-envelope line* of their Hubble diagram of field galaxies on the assumption that the luminosity of M 31 must be matched by at least some galaxies. This elegantly circumvented the problem of Malmquist bias (Fig. 2).

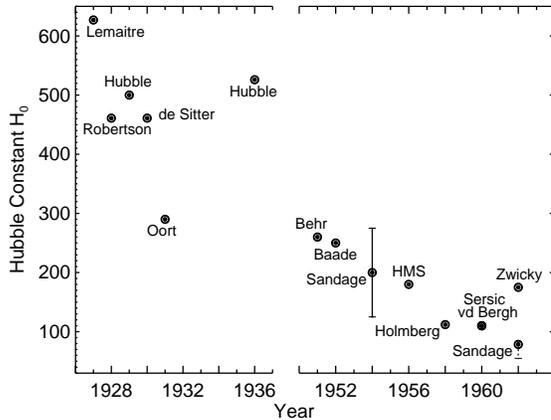


Figure 3: Determinations of H_0 from 1927 to 1962.

The confusion between brightest stars and HII regions was elaborated by Sandage (1958). The corresponding correction together with the correction of Hubble’s photometric scale led him to conclude that the 1936 distance scale was too short by $4^{m}6$ and consequently that $H_0 = 75$. He noted that if the brightest stars had $M_{pg} = -9.5$ (which is now well demonstrated) H_0 would become 55. He also concluded from novae that Hubble’s Local Group distances were more nearly correct, i.e. too small by “only” $2^{m}3$ on average. Sandage’s paper has become a classic for not only having given the first modern values of H_0 , but also because it contains the first physical explanation of the instability strip of Cepheids.

The situation in mid-1961 was summarized by Sandage (1962) at the influential 15th IAU Symposium in Santa Barbara. While he cited values of $H_0 \sim 110$ by Sersic (1960), van den Bergh (1960a), and Holmberg (1958), his own values – based, in addition to Cepheids and brightest stars, on the *size* of HII regions – were $75-82$ and possibly as low as 55. F. Zwicky pleaded in the discussion for $H_0 = 175$ from supernovae. The decrease of H_0 from 1927 to 1962 is illustrated in Fig. 3.

3 Work on H_0 in 1962-1975

A new epoch began with the Cepheid distance of M31 of $(m - M)^0 = 24.20 \pm 0.14$ (Baade & Swope 1963), derived by H. H. Swope after W. Baade’s death from his $200''$ -plates and from H. C. Arp’s photoelectric sequence. (For the history of the time cf. also Sandage 1998, 1999a).

By the same time the “direct” (i.e. non-spectroscopic) staff members at the Mount Wilson and Palomar observatories (W. Baade, E. Hubble, M. Humason, A. Sandage, and others) had accumulated many $200''$ -plates of a few

galaxies outside the Local Group for work on the Cepheids. Hubble and Baade had left their observations to Sandage, who in addition had set up photoelectric sequences around these galaxies, whose faintness and quality has remained unsurpassed until the advent of CCD detectors. Thus there was a unique wealth of observations when I had the privilege to join the project as Sandage’s assistant in 1963.

Although the first Cepheid distance of NGC 2403 to come out of the program confirmed Sandage’s 1962 value (Tammann & Sandage 1968), using the then latest version of the Cepheid P-L relation (Sandage & Tammann 1968), it was criticized as being (much) too large (e.g. Madore 1976; de Vaucouleurs 1978; Hanes 1982). The modern value is actually only marginally smaller (Saha et al. 2005).

The second galaxy of the program, NGC 5457 (M 101), came as a great surprise: its distance was found twice the value of Sandage’s (1962) estimate (Sandage & Tammann 1974b), i.e. $(m - M)^0 = 29.3$. The distance of M 101 and its companions was based on brightest stars, HII region sizes, and van den Bergh’s (1960a) luminosity classes of spiral galaxies, but also heavily on the *absence* of Cepheids down to the detection limit. The faint Cepheids were eventually found with HST, yielding $(m - M)^0 = 29.34$ (Kelson et al. 1996) or 29.18 (Saha et al. 2005). In the mean time the distance had been denounced as being too large (e.g. de Vaucouleurs 1978; Humphreys & Strom 1983).

The new distance of M 101 made clear that the brightest spirals of luminosity class (LC) I are brighter than anticipated and that the luminosity of their brightest stars and the size of their largest HII regions had to be increased. This led immediately to a distance of the Virgo cluster of $(m - M) = 31.45$ (Sandage & Tammann 1974c), – a value probably only slightly too small (cf. Tammann et al. 2002). The ensuing luminosity calibration of LCI spirals could then be applied to a specially selected, *distance-limited* sample of 36 such galaxies, bounded by 8500 km s^{-1} . The conclusion was that $H_0 = 55 \pm 5$ “everywhere” (Sandage & Tammann 1975). The largest contribution to the systematic errors was attributed to the calibration through Cepheids.

In almost half a century from 1927 to 1975 the galaxy distances have increased by roughly a factor of 10. The stretch factor is non-linear, being ~ 2 for the nearby LMC and SMC, but ~ 10 for M 101 and beyond (Fig. 4).

4 H_0 after 1975

Work on H_0 exploded after 1975. The new activity was initiated by G. de Vaucouleurs. Having started with $H_0 = 50$ from brightest globular clusters (de Vaucouleurs 1970), he switched to $H_0 \sim 100 \pm 10$ (de Vaucouleurs 1977; de Vaucouleurs & Bollinger 1979). By assuming rather short local distances and by turning a blind eye to all selection effects, he managed to maintain this value – eventually with strong directional variations – until his last paper on the subject (de Vaucouleurs & Peters 1985).

Old and new methods of distance determinations were employed. They

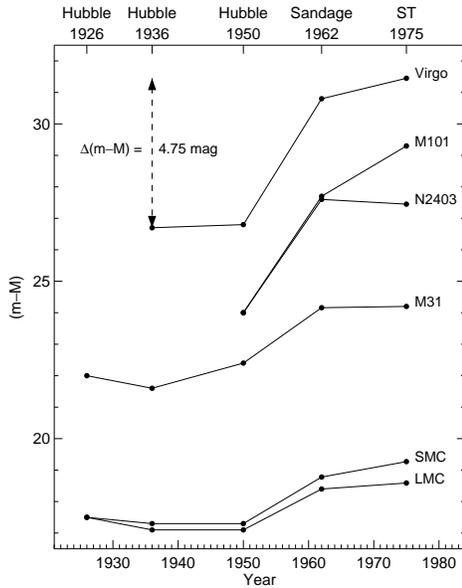


Figure 4: The development in time of some distances of local galaxies as stepping stones for the extragalactic distance scale. (Hubble’s 1950 distances in Holmberg (1950); ST stands for Sandage & Tammann).

may be divided into 1) those using individual objects in galaxies, and 2) those relying on global galaxian properties.

4.1 Individual objects as distance indicators

a) RR Lyr stars. Extensive work on their luminosity calibration in function of metallicity seems now to converge, but there remain some exceptions. Their range is so far confined to the Local Group. For a review see Sandage & Tammann (2006).

b) Cepheids. See Section 5.2.2 and Sandage & Tammann (2006).

c) Brightest stars. The luminosity of brightest stars, Hubble’s classical vehicle, lost much of its grip when it was shown that it depends on the size (luminosity) of the parent galaxy (Sandage & Tammann 1974a).

d) Size of HII regions. The size of the largest HII regions in late-type galaxies was introduced as a distance indicator by Sérsic (1960) and Sandage (1962). Imaging of many galaxies with an H α filter by Sandage extended the distance scale considerably (Sandage & Tammann 1974b), but also here it was found that the size depends on the size of the parent galaxy. The method is not competitive anymore.

e) Globular clusters (GCs). The luminosity of the peak of the bell-shaped luminosity function (LF) of GCs has been proposed as a standard candle (van den Bergh et al. 1985). The method seems attractive because its calibration

depends on the well defined LF of Galactic GCs whose Population II distances are independent of Cepheids. It was employed by several authors (for reviews see Harris 1991; Whitmore 1997; Tammann & Sandage 1999). But the basic *assumption* that the LF was universal is shattered by the fact that some GC color functions and LFs show double peaks, and by doubts that the formation of GCs is a unique process.

f) Novae. After the confusion of novae and supernovae had been lifted by Lundmark (1920), novae played a role as distance indicators in their own right. Instead of the luminosity at maximum, which has a very wide dispersion, the magnitude 15 days after maximum or the luminosity-decline rate relation were used. The independent calibration can come, at least in principle, from expansion parallaxes of Galactic novae (Cohen 1985). The data acquisition of novae is demanding on telescope time and little has been done in recent years.

g) Planetary nebulae (PNe). Following a proposal by Ford & Jenner (1978) also brightest planetary nebulae have been widely used as distance indicators. But the method seems to depend on population size (Bottinelli et al. 1991; Tammann 1993), chemical composition, and age (Méndez et al. 1993); moreover the PNe in NGC 4697 have different LFs depending on their dynamics (Sambhus et al. 2005).

h) The tip of the red-giant branch (TRGB). It was shown by Da Costa & Armandroff (1990) that the TRGB in globular clusters has a fixed absolute *I*-magnitude, irrespective of metallicity. The TRGB has hence been used as a distance indicator by several authors (Lee et al. 1993; Salaris & Cassisi 1997; Madore et al. 1997; Sakai 1999; Karachentsev et al. 2003; Sakai et al. 2004). The method is of great interest since its calibration rests on Population II objects (GCs and RR Lyr stars) and provides an independent test of the Cepheid distance scale. I will return to the point in Section 5.2.2.

i) Supernovae of type Ia (SNe Ia). See Section 5.2.1.

4.2 Global properties of galaxies as distance indicators

a) Luminosity classes (LC) of spiral galaxies. The luminosity of a spiral galaxy correlates with the “beauty” of its spiral structure. Correspondingly they were divided into class I (the brightest) to V (the faintest) by van den Bergh (1960b,c,d) with additional galaxies classified by Sandage & Tammann (1981) and others. The purely morphological LC classification is independent of distance; it yields therefore relative distances which were valuable for many years when velocity distances were suspected to be severely distorted by peculiar and streaming motions. Locally calibrated LC I spirals out to 6000 km s^{-1} from a distance-limited sample were used to derive $H_0 = 56.9 \pm 3.4$ (Sandage & Tammann 1975). Bias-corrected LC distances led Sandage (1999b) to $H_0 = 55 \pm 3$.

b) 21cm-line widths. 21cm (or alternatively optical; see Mathewson et al. 1992; Mathewson & Ford 1996) spectral line widths are a measure of a galaxy’s rotation velocity, if corrected for inclination i , and hence correlate with its

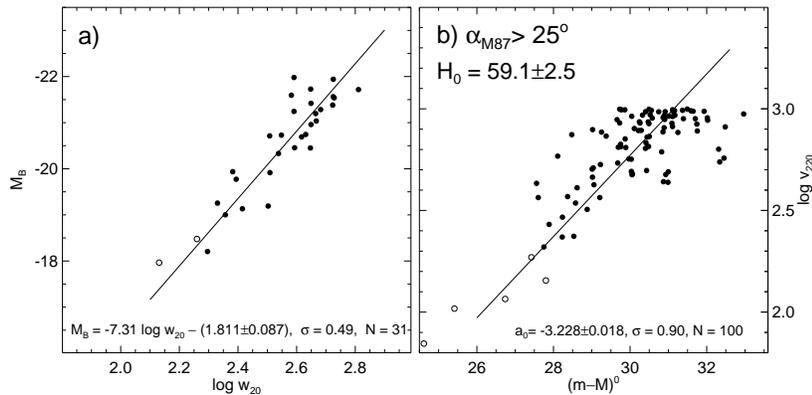


Figure 5: a) Calibration of the TF relation by means of 31 galaxies ($i > 45^\circ$) with known Cepheid distances. The two open circles are companions to M101 which are assumed to be at the distance of M101. The slope is taken from the Virgo cluster. w_{20} is the inclination-corrected line width at the 20% intensity level expressed in km s^{-1} . b) The Hubble diagram of 100 field galaxies within $v_{220} < 1000 \text{ km s}^{-1}$ and known TF distances. The turbulent region with a radius of 25° about the Virgo cluster is omitted. Note the large scatter.

mass and luminosity (Gougenheim 1969). The relation was applied for distance determinations by Tully & Fisher (1977, TF) and many subsequent authors, some of which are listed in Table 2. Several of the solutions for H_0 were dominated by Malmquist bias. The present (2005) calibration of the TF relation rests on 31 galaxies with $i > 45^\circ$ and with known Cepheid distances from Saha et al. (2005); the slope of the relation is taken from a complete sample of 49 inclined spirals in the Virgo Cluster (Fig. 5a). The scatter of $\sigma_m = 0^m 49$ is much larger than can be accounted for by errors of the Cepheid distances; it reflects mainly the large intrinsic scatter of the TF relation.

The calibration can be applied to an almost complete, *distance*-limited sample, as compiled by Federspiel (1999), of 100 inclined spirals with $v_{220} < 1000 \text{ km s}^{-1}$ (for the corrected velocities v_{220} see Section 6). The result in Fig. 5b gives $H_0 = 59.1 \pm 2.5$, but it is disappointing as to the very large scatter, which is much larger than from the calibration in Fig. 5a, even if the turbulent region of radius 25° about the Virgo cluster is omitted. The reason is unclear; it cannot be due to peculiar motions which are much too small (see Section 6). It may be that remaining observational errors of the galaxian parameters contribute to the scatter. In any case the example shows that the TF method is difficult to handle. Open questions remain as to the large corrections for internal absorption, to truncated galaxies and hence to environment, and to the dependence on color and Hubble type. If only the apparently brightest galaxies were considered, arbitrarily large values of H_0 would be the consequence (see Tammann et al. 2002, Fig. 4). The TF is

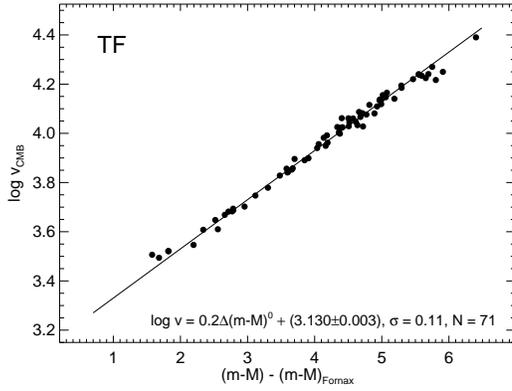


Figure 6: The Hubble diagram of 71 clusters whose distances relative to the Fornax cluster are known from the Tully-Fisher relation. Each point is the mean of about 10 galaxies. (Data from Giovanelli et al. 1997 and Dale et al. 1999).

therefore vulnerable against Malmquist bias, even if the intrinsic scatter was “only” 0^m49 as in Fig. 5a.

An application of the TF calibration in Fig. 5a to a *complete* sample of 49 untruncated spirals with $i > 45^\circ$ in the Virgo cluster (Federspiel et al. 1998) yields a cluster modulus of $(m - M)^0 = 31.62 \pm 0.16$. Again, arbitrarily small values of the distance will emerge if the cluster sample is cut by an apparent-magnitude limit (Kraan-Korteweg et al. 1988, Fig. 6).

Giovanelli et al. (1997) and Dale et al. (1999) have determined TF data for roughly 10 galaxies in each of 51 clusters with $3000 < v_{\text{CMB}} < 25\,000 \text{ km s}^{-1}$. They define a Hubble line (Fig. 6) with very small scatter of 0^m11 , which is even competitive with SNe Ia. Unfortunately the TF calibration of Fig. 5a cannot be directly applied to the cluster sample, because it cannot be assumed that the individual cluster galaxies were selected in the same way as the local galaxies for which Cepheid distances are available; this is a critical condition considering the large intrinsic dispersion if $\sigma \gtrsim 0^m4$ of the TF method (for an opposite view see Giovanelli 1997; Sakai et al. 2000). Instead it is possible to relate all cluster moduli to the modulus of the Fornax cluster. The ensuing equation of the Hubble line is shown at the bottom of Fig. 6. By simple transformation it follows

$$H_0 = -0.2(m - M)_{\text{Fornax}} + (8.130 \pm 0.003). \quad (2)$$

Inserting the Fornax cluster modulus of $(m - M)^0 = 31.54 \pm 0.13$ from Cepheids and SNe Ia (see below) leads to $H_0 = 65.6 \pm 4.1$.

While the distance indicators under 4.2 a), b) involve spiral galaxies, the following three methods use E/S0 galaxies. The disadvantage is that there are no Cepheid distances – nor RR Lyr star or TRGB distances – available

for normal early-type galaxies to set the zero-point of the distance scale. In some cases one may infer an association between an E/S0 galaxy and a spiral with known Cepheid distance, or one may assume that the specific method applies also to the bulges of spiral galaxies. But these cases remain few, while actually many calibrators would be needed in view of the large intrinsic scatter of $\gtrsim 0^m3$, i.e. much larger than that of SNe Ia.

c) Brightest cluster galaxies (BCG). The important potential of BCGs as standard candles to trace the expansion of the Universe was exploited first by Humason (1936; Humason et al. 1956). The work was propelled by Sandage (1967, 1968, 1972, 1973). His papers were the decisive proof for cosmic expansion at a time when many astronomers speculated in view of the large quasar redshifts about a mysterious origin of redshifts. The last paper (Sandage & Hardy 1973) on the subject containing galaxies of moderate redshift lists 72 BCGs with $3500 < v_{\text{CMB}} < 30\,000 \text{ km s}^{-1}$. They define a Hubble line of

$$\log v = 0.2m_{1\text{st}} + (1.364 \pm 0.007) \quad (3)$$

with a scatter of $\sigma_m = 0^m29$. This implies

$$\log H_0 = 0.2M_{1\text{st}} + (6.364 \pm 0.007). \quad (4)$$

The mean absolute magnitude (in their corrected photometric system) of the two BCGs in the Virgo and Fornax clusters is $M_{1\text{st}} = -23^m15$, using the cluster distances from Cepheids and SNe Ia (see below). Hence $H_0 = 54.2 \pm 5.4$.

d) The D_n - σ or fundamental plane method (FP). The correlation of the velocity dispersion σ of E/S0 galaxies with their luminosity was pointed out by Minkowski (1962) and Faber & Jackson (1976). Later the luminosity was replaced by a suitably normalized diameter D_n (Dressler et al. 1987) or by surface brightness (Djorgovski & Davis 1987). The method was extended to the bulges of spiral galaxies by Dressler (1987) who derived $H_0 = 67 \pm 10$. Federspiel (1999) used the great wealth of D_n - σ data by Faber et al. (1989) in two ways. First he derived the modulus difference between the Virgo and Coma cluster to be 3.75 ± 0.20 from 23 Virgo and 33 Coma members. With a Virgo modulus of $(m - M)_{\text{Virgo}}^0 = 31.47 \pm 0.16$ from Section 6 one obtains therefore $(m - M)_{\text{Coma}}^0 = 35.22 \pm 0.26$. Secondly he used an apparent-magnitude-limited subset of 264 early-type, high-quality field and cluster galaxies brighter than 13^m5 to derive a value of H_0 after correcting for Malmquist bias following the method outlined in Federspiel et al. (1994). Beyond $v_{\text{CMB}} = 4000 \text{ km s}^{-1}$ his bias corrections become unreliable because the sample is far from being complete to the apparent-magnitude limit. That Malmquist bias must indeed be a major problem for the D_n - σ and FP methods stems from their intrinsic scatter of $\sigma_m = 0^m36$ as seen in the Coma (Federspiel 1999) and other cluster (Jørgensen et al. 1996). For that reason claims of detected streamings toward the ‘‘Great Attractor’’ just outside 4000 km s^{-1} (Lynden-Bell et al. 1988) are not beyond doubt. – Within $v_{\text{CMB}} = 4000 \text{ km s}^{-1}$ Federspiel’s (1999) analysis yields $H_0 = 57.0 \pm 4.4$ if the Virgo modulus from Section 6 is adopted for the calibration.

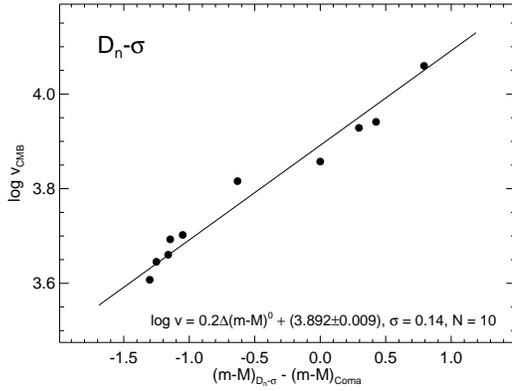


Figure 7: The D_n - σ distances of 9 clusters relative to the Coma cluster. Each point is the mean of about 20 galaxies. (Data from Jørgensen et al. 1996).

Jørgensen et al. (1996) have gained D_n - σ and FP observations of 232 E/S0 galaxies in 10 clusters and determined their mean distances relative to the Coma cluster. Their Hubble diagram is shown in Fig. 7 for the D_n - σ distances, which have a slightly smaller scatter of $\sigma_m = 0^m14$ than their FP distances. The scatter of 0^m14 about the Hubble line of the 9 clusters beyond $v_{\text{CMB}} = 3700 \text{ km s}^{-1}$ is significantly larger than of SNe Ia ($\sigma_m = 0^m10$, see 5.2.1) and cannot mainly be explained by peculiar motions. The Hubble line in Fig. 7 implies

$$\log H_0 = -0.2(m - M)_{\text{Coma}}^0 + (8.892 \pm 0.009), \quad (5)$$

which yields with $(m - M)_{\text{Coma}}^0$ from above $H_0 = 70.4 \pm 9.0$.

An interesting by-product of the equation at the bottom of Fig. 7 is the mean recession velocity of the Coma cluster freed from all peculiar velocities and streamings, if one assumes that the peculiar motions of the 9 clusters beyond 3700 km s^{-1} average out. The zero-point of the relative distance scale should be reliable to within 0^m05 because 44 D_n - σ distances are available for Coma. From this follows an unperturbed velocity of $v_{\text{Coma}} = 7800 \pm 200 \text{ km s}^{-1}$.

e) Surface brightness fluctuations (SBF). This method has been introduced by Tonry & Schneider (1988) and extensively used for E/S0 galaxies (Tonry et al. 2001). The size of the fluctuations shows little dependence on metallicity if measured in the infrared; the dependence on stellar population is compensated by allowing for the color ($V-I$). SBF distances of four recent investigations, based on observations with HST, are plotted in a Hubble diagram in Fig. 8. The distance zero-point depends entirely on Cepheid distances, either of up to six spirals whose bulges are treated like an E/S0 galaxy or/and of 1-5 aggregates containing E/S0's as well as spirals with Cepheid distances. The relative small scatter of $\sigma_m = 0^m26$ beyond 3000 km s^{-1} , – yet significantly

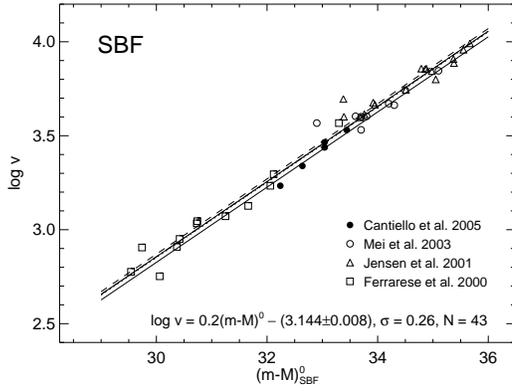


Figure 8: The Hubble diagram with SBF distances from different authors. Objects in the turbulent region within 25° from the Virgo cluster center are not shown. The mean Hubble line suggests $H_0 = 71.8$, but the zero-point calibration remains unreliable.

larger than for SNeIa in dust-free parent galaxies ($\sigma_m = 0^m10$, see 5.2.1), – shows that the method works in principle. But the result on H_0 is paradoxical being 15% larger even locally than from the direct evidence from Cepheids (see Sec. 6, Fig. 12b). If Cepheids are trusted at all, a Cepheid-calibrated distance indicator (SBF) must on average reproduce the distance scale of the Cepheids. There remains therefore a problem with the zero-point calibration; either because the bulges of spirals have different stellar populations as E/S0's, a possibility pointed out by Ferrarese et al. (2000), or because of unaccounted dust in spiral bulges. – At larger distances also Malmquist bias may play a rôle in view of $\sigma_m = 0^m26$. Many of the distant galaxies are brightest cluster galaxies, and it may not be warranted to extrapolate the SBF-magnitude relation from local calibrators – some of them being only spiral bulges! – to galaxies with $M \lesssim -23^m0$. The conclusion is that SBFs yield *relative* distances within $\sim 13\%$, but that they are not (yet) to be used for the determination of H_0 . The Coma cluster is not useful for the calibration at large distance because only three member galaxies have SBF measurement and the cluster distance itself has a large error.

An overview of the present determinations of H_0 outside $v_{\text{CMB}} > 3000 \text{ km s}^{-1}$ is given in Table 1; the value from SNeIa from 5.2.3 is anticipated for comparison.

The mean value of $H_0 = 60.7 \pm 2.5$ from the first five lines in Table 1 is in statistical agreement with the value from SNeIa in 5.2.3. The latter have the decisive advantage of having very small scatter (0^m10) and being hence insensitive to Malmquist bias; it rests in addition on a solid zero-point from 10 direct Cepheid distances.

Table 1: Present determinations of H_0 reaching out to $v_{\text{CMB}} > 3000 \text{ km s}^{-1}$.

Method intrinsic	σ_m km s^{-1}	range	calibration ¹⁾	H_0 ²⁾
TF	0.45	25 000	Fornax	65.6 ± 4.1
BCG	0.30	30 000	Virgo + Fornax	54.2 ± 5.4
D_n - σ	0.36	4 000	Virgo	57.0 ± 4.4
D_n - σ	0.36	10 000	Coma	70.4 ± 9.0
SBF	0.26	10 000	Cepheid dist.	(71.6)
SNe Ia	0.10	30 000	10 Cepheid dist.	62.3 ± 1.3

¹⁾ For easier comparison all underlying Cepheid distances are taken from Saha et al. (2005)

²⁾ The systematic error of the Cepheid distance scale is not included

4.3 Various determinations of H_0 after 1975

The above distance indicators have been used in various combinations to derive values of H_0 . Many authors have contributed; a representative subset has been compiled in Table 2. The resulting values of H_0 since 1975 are plotted in Fig. 9 against the year of publication.

Table 2: Values of H_0 from 1974 – 2005.

Year	H_0	Code	Reference
(a) various methods, corr. for bias [●]			
1974	56	ST	Sandage, A., & Tammann, G. A. 1974, ApJ 194, 223
1974	57	ST	Sandage, A., & Tammann, G. A. 1974, ApJ 194, 559
1975	57	ST	Sandage, A., & Tammann, G. A. 1975, ApJ 196, 313; 197, 265
1977	52.5	T	Tammann, G. A. 1977, in Redshifts and the Expansion of the Universe, 43
1982	50	ST	Sandage, A., & Tammann, G. A. 1982, ApJ 256, 339
1988	69	vdB	van den Bergh, S. 1988, in The Extragalactic Distance Scale, ASP Conf. Ser. 4, 375
1988	56	T	Tammann, G. A. 1988, in The Extragalactic Distance Scale, ASP Conf. Ser. 4, 282
1988	55	Te	Terndrup, D. M. 1988, in The Extragalactic Distance Scale, ASP Conf. Ser. 4, 211
1990	71	Go	Gouguenheim, L., et al. 1990, in Proc. XXIVth Moriond Meeting, 3
1990	52	ST	Sandage, A., & Tammann, G. A. 1990, ApJ 365, 1
1995	57	ST	Sandage, A., & Tammann, G. A. 1995, ApJ 446, 1
1996	56	S	Sandage, A. 1996, AJ 111, 1
1996	50	S	Sandage, A. 1996, AJ 111, 18
1996	81	vdB	van den Bergh, S. 1996, PASP 108, 1091
1997	52.5	G	Goodwin, S. P., Gribbin, J., & Hendry, M. A. 1997, AJ 114, 2212
1997	55	T	Tammann, G. A., & Federspiel, M. 1997, in The Extragalactic Distance Scale, ed. M. Livio (Cambridge Univ. Press), 137
1998	60	Pt	Paturel, G., et al. 1998, A&A 339, 671
1999	55	S	Sandage, A., 1999, ApJ 527, 479
2000	68	M	Mould, J. R., et al. 2000, ApJ 529, 786
2001	55	T	Tammann, G. A., Reindl, B., & Thim, F. 2001, in Cosmology and Particle Physics, AIP Conf. Proc. 555, 226

Table 2: (Continued)

Year	H_0	Code	Reference
2002	58	S	Sandage, A. 2002, AJ 123, 1179
2002	59.2	T	Tammann, G. A., et al. 2002, in A New Era in Cosmology, ASP Conf. Proc. 283, 258
2002	56.9	T	Tammann, G. A., & Reindl, B. 2002, in The Cosmological Model, XXXVIIth Moriond Ap. Meeting, 13
(b) various methods, <i>not</i> corr. for bias [c]			
1972	100	deV	de Vaucouleurs, G. 1972, in External Galaxies and Quasi-Stellar Objects, IAU Symp. 44, 353
1976	75	deV	de Vaucouleurs, G. 1976, ApJ 205, 13
1977	85	deV	de Vaucouleurs, G. 1977, in Redshifts and the Expansion of the Universe, 301
1978	95	deV	de Vaucouleurs, G. 1978, in The Large Scale Structure of the Universe, IAU Symp. 79, 205
1981	96	deV	de Vaucouleurs, G., & Peters, W. L. 1981, ApJ 248, 395
1986	109	deV	de Vaucouleurs, G., & Peters, W. L. 1986, ApJ 303, 19
1986	99	deV	de Vaucouleurs, G., & Corwin, H. G. 1986, ApJ 308, 487
1986	95	deV	de Vaucouleurs, G. 1986, in Galaxy distances and deviations from universal expansion, eds. B. F. Madore & R. B. Tully, (Dordrecht: Reidel), 1
1993	85	deV	de Vaucouleurs, G. 1993, ApJ 415, 10
1993	90	Tu	Tully, R. B. 1993, in Proc. Nat. Acad. Sci. 90, 4806
1997	81	Gz	Gonzales, A. H., & Faber, S. M. 1997, ApJ 485, 80
1997	73	M	Mould, J. R., et al. 1997, in The Extragalactic Distance Scale, ed. M. Livio (Cambridge Univ. Press), 158
2001	72	Fr	Freedman, W. L., et al. 2001, ApJ 553, 47
(c) SNe Ia [◊]			
1982	50	ST	Sandage, A., & Tammann, G. A. 1982, ApJ 256, 339
1988	59	Br	Branch, D. 1988, in The Extragalactic Distance Scale, ASP Conf. Ser. 4, 146
1990	46.5	TL	Tammann, G. A., & Leibundgut, M. 1990, A&A 236, 9
1994	52	SN	Saha, A., et al. 1994, ApJ 425, 14
1995	52	SN	Saha, A., et al. 1995, ApJ 438, 8
1995	56.5	SN	Tammann, G. A., & Sandage, A. 1995, ApJ 452, 16
1995	71	Pi	Pierce, M. J., & Jacoby, G. H. 1995, AJ 110, 2885
1996	56.5	SN	Saha, A., et al. 1996, ApJ 466, 55
1996	63.1	H	Hamuy, M., et al. 1996, AJ 112, 2398
1997	56	SN	Saha, A., et al. 1997, ApJ 486, 1
1999	60	SN	Saha, A., et al. 1997, ApJ 522, 802
1999	62.9	TB	Tripp, R., & Branch, D. 1999, ApJ 525, 209
1999	63.9	Su	Suntzeff, N. B., et al. 1999, ApJ 500, 525
1999	63.3	Ph	Phillips, M. M. 1999, AJ 118, 1766
1999	64.4	J	Jha, S., et al. 1999, ApJS 125, 73
2000	68	G	Gibson, B. K., et al. 2000, ApJ 529, 723
2000	58.5	SN	Parodi, B. R., et al. 2000, ApJ 540, 634
2001	71	Fr	Freedman, W. L., et al. 2001, ApJ 553, 47
2001	58.7	SN	Saha, A., et al. 2001, ApJ 562, 314
2004	71	A	Altavilla, G., et al. 2004, MNRAS 349, 1344
2005	73	R	Riess, A. G., et al. 2005, ApJ 627, 579
2006	62.3	SN	Tammann, G. A., et al., ApJ, to be published
(d) Tully-Fisher, corr. for bias [▲]			
1976	50	ST	Sandage, A., & Tammann, G. A. 1976, ApJ 210, 7
1997	55	Th	Theureau, G., et al. 1997, A&A 322, 730
1999	58	F	Federspiel, M. 1999, Ph.D. Thesis, Univ. of Basel

Table 2: (Continued)

Year	H_0	Code	Reference
1999	53	E	Ekholm, T., et al. 1999, A&A 347, 99
2000	55	Th	Theureau, G. 2000, in XIX Texas Symposium, eds. E. Aubourg et al. Mini-Symp. 13/12
2002	65	He	Hendry, M. A. 2002, in New Era in Cosmology, eds. T. Shanks, & N. Metcalfe, ASP Conf. Ser. 283, 258
(e) Tully-Fisher, <i>not</i> corr. for bias [Δ]			
1977	82	Tu	Tully, R. B., & Fisher, J. R. 1977, in Redshifts and the Expansion of the Universe, 95
1980	95	Aa	Aaronson, M., et al. 1980, ApJ 239, 12
1984	91	Bo	Bothun, G.D., et al. 1984, ApJ 278, 475
1986	90	Aa	Aaronson, M., et al. 1986, ApJ 302, 536
1988	85	Pi	Pierce, M. J., & Tully, R. B. 1988, ApJ 330, 579
1988	85	H	Huchra, J. P. 1988, in The Extragalactic Distance Scale, ASP Conf. Ser. 4, 257
1994	86	Pi	Pierce, M. J. 1994, ApJ 430, 53
1997	70	Gi	Giovanelli, R. 1997, in The Extragalactic Distance Scale, ed. M. Livio (Cambridge Univ. Press), 113
2000	77	Tu	Tully, R. B., & Pierce, M. J. 2000, ApJ 533, 744
2000	81	R	Rothberg, B., et al. 2000, ApJ 533, 781
2000	71	Sk	Sakai, S., et al. 2000, ApJ 529, 698
(f) D_n - σ , fundamental plane [+]			
1987	67	D	Dressler, A. 1987, ApJ 317, 1
1999	52	F	Federspiel, M. 1999, Ph.D. Thesis, Univ. of Basel
2000	78	K	Kelson, D. D., et al. 2000, ApJ 529, 768
(g) globular clusters[\times]			
1979	80	Hn	Hanes, D. A. 1979, MNRAS 188, 901
1988	61	Hs	Harris, W. E. 1988, in The Extragalactic Distance Scale, ASP Conf. Ser. 4, 231
1993	85	deV	de Vaucouleurs, G. 1993, ApJ 415, 33
1995	78	W	Whitmore, B. C., et al. 1995, ApJ 454, 73
1996	68	B	Baum, W. A., et al. 1996, A&AS 189, 1204
1997	82	W	Whitmore, B. C. 1997, in The Extragalactic Distance Scale, ed. M. Livio (Cambridge Univ. Press), 254
2000	69	K	Kavelaars, J. J., et al. 2000, ApJ 533, 125
(h) planetary nebulae [o]			
1990	87	Ja	Jacoby, G. H., et al. 1990, ApJ 356, 332
1991	77	Bo	Bottinelli, L., et al. 1991, ApJ 252, 550
1993	75	Mc	McMillan, R., et al. 1993, ApJ 416, 62
2002	78	Ci	Ciardello, R., et al. 2002, ApJ 577, 31
(i) surface brightness fluctuations [o]			
1989	88	To	Tonry, J. L., et al., 1989, ApJ 346, 57
1997	81	To	Tonry, J. L. 1997, in The Extragalactic Distance Scale, ed. M. Livio (Cambridge Univ. Press), 297
1998	82	La	Lauer, T. R., et al. 1998, ApJ 499, 577
1999	74	Bl	Blakeslee, J. P., et al. 1999, ApJ 527, 73
1999	87	Je	Jensen, J. B., et al. 1999, ApJ 510, 71
2000	77	To	Tonry, J. L., et al. 2000, ApJ 530, 625
2001	73	Aj	Ajhar, E. A., et al. 2001, ApJ 559, 584

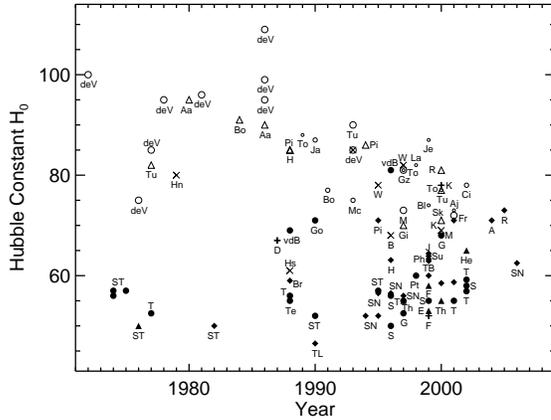


Figure 9: Various values of H_0 since 1975. Different symbols indicate different methods of distance determinations. Open symbols indicate when H_0 is based on apparent-magnitude-limited samples; closed symbols stand for bias-free or bias-corrected samples.

5 HST and H_0

With the advent of HST two major campaigns were started for the determination of H_0 .

5.1 The HST Key Project on the Extragalactic Distance Scale

The original program was to observe Cepheids in many inclined spirals in order to provide a calibration for the I -band TF relation (Aaronson & Mould 1986); at the time the authors still favored a value of $H_0 = 90$. Later the Cepheid distances were planned (Kennicutt et al. 1995) to also calibrate the LF of PNe and the expanding-atmosphere parallaxes of SNe II, of novae, and of the peak of the LF of GCs. Surprisingly the authors made only cursory reference to the problem of Malmquist bias. The program team, consisting of 26 collaborators provided 19, i.e. almost half of all published Cepheid distances. The distances were based on the P-L relation of 22 LMC Cepheids and a zero-point set at $(m - M)_{\text{LMC}}^0 = 18.50$ (Madore & Freedman 1991). In a first summary paper Mould et al. (2000) concluded from the TF and SBF methods, from SNe Ia and, now also from the FP method that $H_0 = 68 \pm 6$, if they made allowance for high-metallicity, (long-period) Cepheids being somewhat brighter than their LMC counterparts. Unfortunately Freedman et al. (2001) raised the result to $H_0 = 72 \pm 8$ on the basis of an interim P-L relation (Udalski et al. 1999) which is now untenable.

5.2 The HST Project for the Luminosity-Calibration of SNe Ia

A small group of astronomers (A. Saha, F. D. Macchetto, N. Panagia, I. and A. Sandage as PI) proposed to observe Cepheids with HST in galaxies which had produced a well observed SN Ia. The results for 8 galaxies were published; 4 additional ones came from external sources (Turner et al. 1998; Tanvir et al. 1999; Macri et al. 2001; Riess et al. 2005). Two out of the 12 SNe Ia are spectroscopically peculiar and were excluded, leaving 10 Cepheid distances for the calibration of normal SNe Ia. The program has only recently been completed because (1) the WFPC2 on HST was to be recalibrated (Saha et al. 2005), and (2) unexpected complications were found with the P-L relation of Cepheids (see below Sec. 5.2.2). The route to H_0 was described in five papers (Tammann et al. 2003; Sandage et al. 2004, 2006; Reindl et al. 2005; Saha et al. 2005), of which only a summary is given here.

5.2.1 The Hubble diagram of SNe Ia

The first Hubble diagram of SNe Ia was shown by Kowal (1968). Its large dispersion was steadily decreased by subsequent authors. By 1979 SNe Ia had emerged as so reliable standard candles that it could be proposed to observe them at large redshifts ($z \gtrsim 0.5$) for a determination of Λ (Tammann 1979). It is well known that this has become possible since; how much easier must it be to use SNe Ia at small redshifts for a determination of H_0 ! – if only their luminosity calibration is realized.

There are now 124 SNe Ia nearer than $30\,000\text{ km s}^{-1}$ with known B , V and in most cases I magnitudes at maximum as well as decline rates Δm_{15} (the decline in mag over the first 15 days past B_{max}). Excluding 13 spectroscopically peculiar objects leaves 111 normal SNe Ia. Their magnitudes are corrected for Galactic and internal absorption (Reindl et al. 2005). The internal reddening is determined by adopting the intrinsic colors $(B-V)^0$ and $V-I)^0$ – and their non-negligible dependence on Δm_{15} – from 21 SNe Ia in (almost) dust-free E/S0 galaxies. The absorption-corrected absolute magnitudes M_{BVI}^0 , calculated from velocity distances, correlate with the Hubble type of the parent galaxy, SNe Ia in early-type galaxies being fainter. This dependence on Hubble type can empirically be removed by normalizing the magnitudes to a standard value of the decline rate, say $\Delta m_{15} = 1.10$. Also the slight dependence of the luminosity on $(B-V)^0$ is removed by normalizing to the color at $\Delta m_{15} = 1.1$ [$(B-V)_{1.1}^0 = -0.024$]. The resulting magnitudes m_{BVI}^{corr} can be plotted in a Hubble diagram; as an example m_V^{corr} is shown in Fig. 10. A fiducial sample of 62 normal SNe Ia with $3000 < v_{\text{CMB}} < 20\,000\text{ km s}^{-1}$, i.e. in the ideal range to calibrate the large-scale value of H_0 , define a Hubble line of

$$\log v = 0.2m_{\lambda}^{\text{corr}} + C_{\lambda}, \quad (6)$$

with $C_B = 0.693 \pm 0.004$, $C_V = 0.688 \pm 0.004$, $C_I = 0.637 \pm 0.004$. The solution for the intercept C_{λ} is very robust against choosing different SN subsets (see

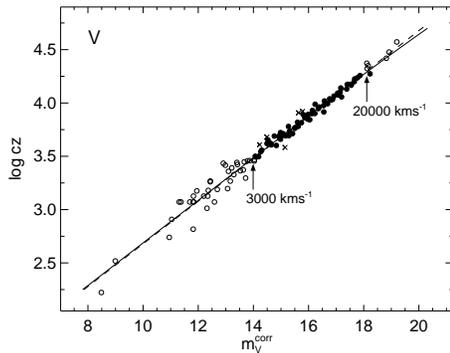


Figure 10: The Hubble diagram in V of 111 normal SNe Ia. The objects outside the indicated velocity range are shown as open symbols; at low velocities the scatter increases because of the influence of peculiar velocities. The slightly curved Hubble line for $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ is a fit to only the black symbols; the crosses are not considered for the fit. The dashed line holds for an $\Omega_T = 0$ Universe.

Reindl et al. 2005, Table 9). The small scatter of $\sigma_m = 0^m15$ – smaller than for any other known individual objects – makes SNe Ia ideal standard candles. In fact much of the scatter is driven by errors of the internal absorption correction, because the 21 SNe Ia in E/S0’s have a scatter in I of only 0^m10 ! Transforming eq. (6) yields

$$\log H_0 = 0.2M_\lambda^{\text{corr}} + C_\lambda. \quad (7)$$

In order to obtain H_0 it remains “only” to calibrate M_λ^{corr} for some nearby SNe Ia with known Cepheid distances. It may be noted that the error of C_λ is so small, that the statistical error of H_0 will essentially depend on only the error of M_λ^{corr} .

5.2.2 Cepheid distances of galaxies with SNe Ia

The determination of Cepheid distances has become much more complicated since it has been realized that the P-L relation is *not* universal. In particular the relations in the Galaxy and in LMC are significantly different (Tammann et al. 2002). The Galactic P-L relation in BVI is quite well defined by 33 Cepheids in open clusters (with a zero-point at $(m - M)_{\text{Pleiades}}^0 = 5^m61$) and 36 Cepheids with moving-atmosphere (BBW) parallaxes by Fouqué et al. (2003) and a few others (see also Ngeow & Kanbur 2004). The P-L relation in BVI of LMC rests on 593 very well observed Cepheids from the OGLE program (Udalski et al. 1999) and 97 bright Cepheids from various sources as well as an adopted zero-point of $(m - M)_{\text{LMC}}^0 = 18.54$. Long-period Galactic Cepheids with a mean metallicity of $[\text{O}/\text{H}] = 8.60$ are *brighter* than their LMC counterparts with $[\text{O}/\text{H}] = 8.36$. The details of the two different P-L

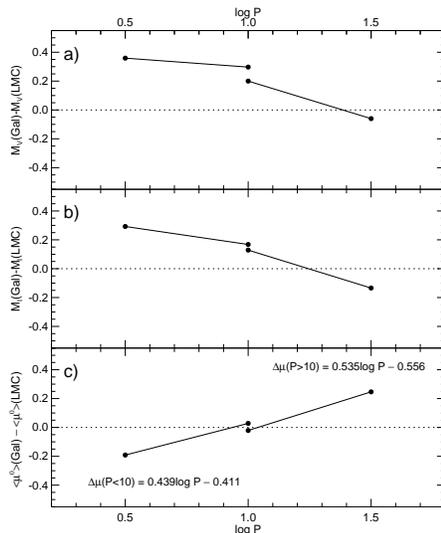


Figure 11: a) The difference between the absolute V magnitude of a Cepheid determined once from the Galactic and once from the LMC P-L relation. b) Same for I magnitudes. c) The distance difference from eq (8) if once M_V and M_I are taken from the Galactic and once from the LMC P-L relation. All differences are plotted in function of $\log P$.

relations are laid out in Tammann et al. (2003) and Sandage et al. (2004). An important feature of the LMC P-L relation is that its slope breaks at $P = 10^d$ (see also Ngeow et al. 2005), which is not seen in the Galaxy (and SMC).

The crux is that the two different P-L relations yield two different distances for every galaxy. The problem is aggravated when only V and I magnitudes are used, as in the case of the HST observations, to determine the true distance modulus *and* the mean internal absorption of the Cepheids. In that case the true and apparent moduli are connected by

$$(m - M)^0 = 2.52(m - M)_I - 1.52(m - M)_V. \quad (8)$$

(The coefficients depend on the adopted absorption-to-reddening ratio \mathcal{R}). Fig. 11 shows how the magnitudes M_V and M_I as well as the true moduli differ in function of period when once the Galactic P-L relation is used and once the one from LMC. The moduli of the former are larger by up to 0^m25 at $\log P = 1.5$.

Only a small part of the P-L relation differences can be explained as the line blanketing effect of the metals, but the main effect is that LMC Cepheids are hotter than Galactic Cepheids at given period or given luminosity. The reason is unknown at present. But Saha et al. (2005) have made the *assumption* that the whole difference is a metallicity effect. Consequently they have derived Cepheid distances of 37 galaxies by interpolating (and slightly ex-

trapolating) their distances from Galactic and LMC P-L relations according to their metallicity as measured by $[O/H]$. The ensuing metallicity corrections are somewhat larger than proposed by Kennicutt et al. (1998) and Sakai et al. (2004), but they are justified by several comparisons with external data; for instance the adopted Cepheid distances of nine galaxies for which also independent, metal-insensitive TRGB distances are available (Sakai et al. 2004) show no systematic trend with $[O/H]$. Also the resulting SN Ia luminosities do not show a significant correlation with the metallicity of their parent galaxies. Finally it may be noted that the metal-rich (inner) and metal-poor (outer) Cepheids in M 101 give the same distance to within 0^m01 .

5.2.3 The value of H_0

As mentioned before there are 10 normal SNe Ia in galaxies with Cepheid distances. The absorption-corrected, normalized magnitudes m_{BVI}^{corr} of these SNe Ia are derived in exactly the same way – and this is an important point – as for the distant SNe Ia which define the Hubble diagram in Fig. 10 (Reindl et al. 2005). The metallicity-corrected distances of the 10 SN Ia parent galaxies are derived by Saha et al. (2005). Combining the magnitudes m_{BVI}^{corr} with the corresponding distances yields immediately the absolute magnitudes; the weighted means become $M_B^{\text{corr}} = -19.49 \pm 0.04$, $M_V^{\text{corr}} = -19.46 \pm 0.04$, and $M_I^{\text{corr}} = -19.22 \pm 0.05$. By inserting the absolute magnitudes with their appropriate intercepts C_λ (eq. 6) into eq. (7) one finds $H_0(B) = 62.4 \pm 1.2$, $H_0(V) = 62.4 \pm 1.5$, and $H_0(I) = 62.1 \pm 1.4$, or the mean for scales of the order of $20\,000 \text{ km s}^{-1}$

$$H_0 = 62.3 \pm 1.3 \quad (\text{random error}). \quad (9)$$

The systematic error of this result has been discussed in some detail by Sandage et al. (2006) and estimated to be ± 5.3 , most of which is caused by the non-uniqueness of the P-L relation of Cepheids and the closely related question of the metallicity correction of Cepheid distances.

6 The Local Value of H_0 and the Random Motions of Field Galaxies

For a number of “local” galaxies with $v_{220} < 2000 \text{ km s}^{-1}$ Cepheid and/or SN Ia distances are available. Excluding members of the Virgo and Fornax clusters and four nearby galaxies with $(m - M)^0 < 28.2$ leaves 34 galaxies with at least one distance determination. Their distance-calibrated Hubble-diagram shows a very large scatter of $\sigma_m = 1^m0$, which can only be due to peculiar velocities. It is reduced to $\sigma_m = 0^m46$ if the 12 galaxies are omitted whose distance from the Virgo cluster (M 87) is $< 25^\circ$ (Fig. 12a). Clearly a region of 25° ($\sim 8 \text{ Mpc}$) radius about the Virgo cluster is characterized by much larger turbulent motions than the “normal” field. The scatter in the field is further reduced to $\sigma_m = 0^m32$ if the velocities v_0 (corrected to the

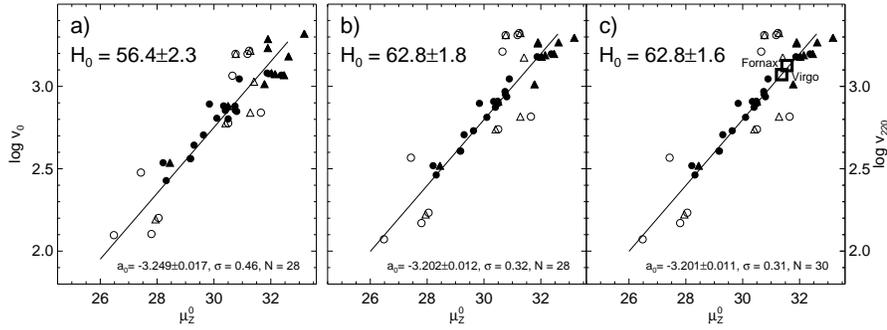


Figure 12: The local distance-calibrated Hubble diagram of 34 galaxies with $v_{220} < 2000 \text{ km s}^{-1}$ for which 27 Cepheid distances (dots) and 16 SN Ia distances (triangles) are available. Galaxies within 25° from the Virgo cluster or with $(m - M)^0 < 28.2$ are shown as open symbols. The Hubble line is a fit to only the closed symbols. a) using velocities v_0 corrected to the barycenter of the Local Group. b) using velocities v_{220} corrected for Virgocentric infall. c) same as b) but with the Virgo and Fornax clusters added.

centroid of the Local Group; Yahil et al. 1977) are replaced by v_{220} . The v_{220} velocities are corrected for a selfconsistent Virgocentric infall model with a local Virgocentric velocity vector of 220 km s^{-1} (Yahil et al. 1980; Tammann & Sandage 1985; Kraan-Korteweg 1986). This model finds here support from the unexpectedly small scatter in Fig. 12b, where the v_{220} velocities are used.

In Fig. 12c also the Virgo and Fornax clusters are plotted with their mean Cepheid and SN Ia distances ($\langle m - M \rangle_{\text{Virgo}}^0 = 31.47 \pm 0.16$, $\langle v_{220} \rangle = 1179 \text{ km s}^{-1}$ and $\langle m - M \rangle_{\text{Fornax}}^0 = 31.56 \pm 0.13$, $\langle v_{220} \rangle = 1338 \text{ km s}^{-1}$; see Sandage et al. 2006). They fit on the Hubble line well within the errors.

The resulting value of $H_0(\text{local}) = 62.8 \pm 1.6$ is undistinguishable from the large-scale value. This does not mean that the expansion is blind toward density fluctuations, because the gravitational effect of the Virgo cluster complex has been eliminated by subtracting the Virgocentric flow model.

The small scatter of 0^m32 in Fig. 12b of the field galaxies outside the 25° circle puts strong upper limits on the size of the peculiar motions, i.e. $\partial v/v = 0.16$ even without allowing for distance errors. The typical peculiar velocity of a galaxy at say 1000 km s^{-1} is therefore $< 160 \text{ km s}^{-1}$. – Also the peculiar motions of more distant field galaxies are restricted by SNe Ia. The 20 SNe Ia in E/S0 galaxies (and hence little internal absorption) with $5000 < v_{\text{CMB}} < 20000 \text{ km s}^{-1}$ scatter about the Hubble line, as stressed before, by only 0^m10 (in I magnitudes, Reindl et al. 2005). Some of this scatter must be due to photometric errors and to the intrinsic dispersion of the normalized SN Ia magnitudes; $\partial v/v = 0.05$ or $v_{\text{pec}} = 300 \text{ km s}^{-1}$ at a distance of 6000 km s^{-1} are therefore generous upper limits.

7 Concluding Remarks

In general astronomical distances depend on objects whose distances are already known and ultimately, with a few exceptions, on trigonometric parallaxes and hence on the AU. But methods of determining distances from the physics or geometry of some objects, without recourse to any other astronomical distance, are gaining increasing weight. Already the moving-atmosphere (BBW) method contributes to the calibration of the Galactic P-L relation of Cepheids. The single, intrinsically accurate water maser distance of NGC 4258 (Herrnstein et al. 1999) does not yet suffice for an independent calibration of the P-L relation (see Saha et al. 2005). The recently improved expanding-atmosphere distance of SN II 1999em (Baron et al. 2004) agrees well with the Cepheid distance of its parent galaxy NGC 1637. Nadyozhin's (2003) plateau-tail method for SNe IIP yields $H_0 = 55 \pm 5$ on the assumption that the ^{56}Ni mass equals the explosion energy. Models of SNe Ia yield $M_{\text{bol}} \approx M_V = -19.5$ (Branch 1998, for a review) in fortuitous agreement with the empirical value of $M_V = -19.46$.

Much promise to determine H_0 accurately lies in the Sunyaev-Zeldovich (SZ) effect and in gravitationally lensed quasars; extensive work has gone into both methods. The SZ effect yields typical values of $H_0 = 60 \pm 3$, yet the systematic error is still $\sim \pm 18$ (Carlstrom et al. 2002 for a review; see also e.g. Udomprasert et al. 2004; Jones et al. 2005). Results from lensed quasars lie still in a wide range of $48 < H_0 < 75$ (e.g. Saha & Williams 2003; Koopmans et al. 2003; Kochanek & Schechter 2004; York et al. 2005).

A strong driver to determine H_0 as accurately as possible comes from the CMB. The interpretation of its Fourier spectrum depends on at least twelve free parameters, several of which cannot be determined elsewhere. A simultaneous solution for all twelve parameters yields $H_0 = 66 \pm 7$ (Rebolo et al. 2004). It would bring important progress for the understanding of the CMB spectrum if an independently determined value of H_0 could be used as a reliable prior.

The value of $H_0 = 62.3$ corresponds to an expansion age of 15.1 Gyr in a flat Λ CDM model with $\Omega_\Lambda = 0.7$. This gives a sufficient time frame for the oldest ages in the Galaxy. Stellar-evolution models give for M107 14.0 ± 2.8 (Chaboyer et al. 2000) and for M92 13.5 Gyr (VandenBerg et al. 2002). Models of the chemical evolution of the Galaxy yield ages of the actinides of 12.4–14.7 Gyr (Thielemann et al. 1987) or a U/Th age of 14.5 ± 2.5 (Dauphas 2005). The emphasis has shifted over the last years to the Th/Eu dating of ultra-metal-poor giants. Typical results lie between 14.2 ± 3.0 and 15.6 ± 4.0 Gyr (Cowan et al. 1999; Westin et al. 2000; Truran et al. 2001; Sneden et al. 2003). The ages are to be increased by the gestation time of the relevant objects; some galaxies may also have started their star formation before the Galaxy did. The present age determinations are not yet sufficiently accurate to set stringent limits on H_0 , but the essential point is that none of these ages are significantly larger than allowed for by the expansion age.

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